

# UNCERTAINTIES DUE TO GRAIN SIZE ISSUES IN RESIDUAL STRESS DETERMINATIONS USING NEUTRON DIFFRACTION

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With thanks to: René V. Martins, Michael Hofmann, Joana Rebelo Kornmeier,  
Shanmukha Moturu, Mirko Boin, Anastasius Youtsos and Carsten Ohms

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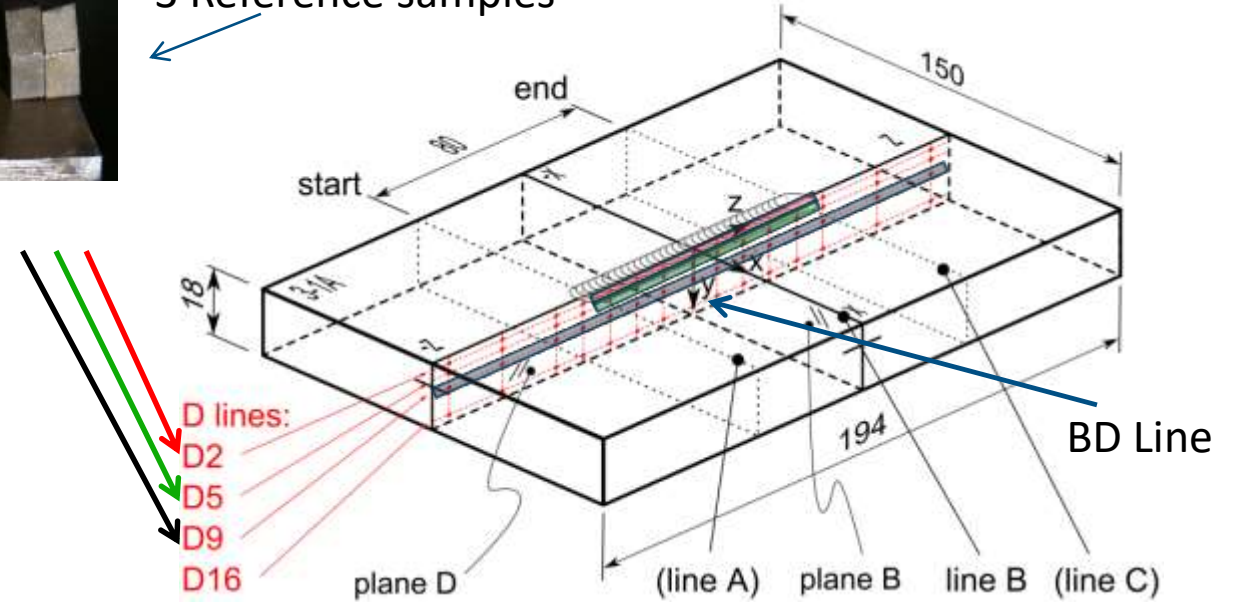
Workshop on the Assessment  
of Residual Stresses in Welds

# TG4: The perfect specimen to study

Main sample



3 Reference samples



Picture of the TG4 'main sample' 3-1A in-situ in the residual stress diffractometer E3 at the HZB, Berlin with an input slit optic of  $3 \times 3 \text{ mm}^2$ .

Grain size varies along BD line  
Oscillation (stepwise or continuous) can reduce the 'grain-size effect'

This is what we need

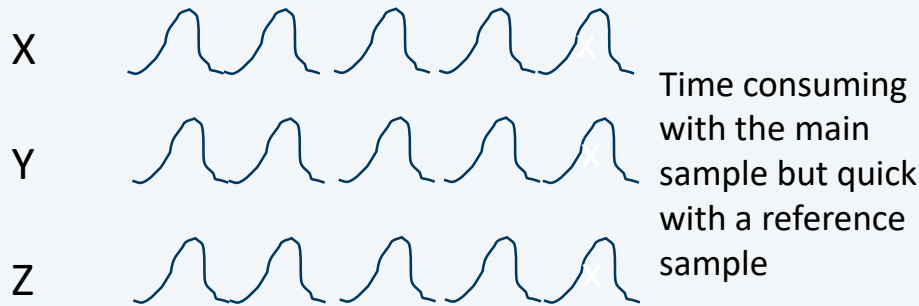
$$\rightarrow u(2\theta) \approx (u(2\theta_{fitting})^2 + u(2\theta_{grain})^2)^{1/2} \rightarrow u(\epsilon) \rightarrow u(\sigma)$$

### Multiple measurement

For one stress determination (e.g 5 measurements/more)

Peaks of 1 unit time (for example)

Sample different sets of grains  
(oscillations would loose information)



Information available:

$$u(2\theta)$$

≈ standard deviation of the 2θ fits values.

$$u(2\theta_{fitting}) \text{ and } u(2\theta_{grain})$$

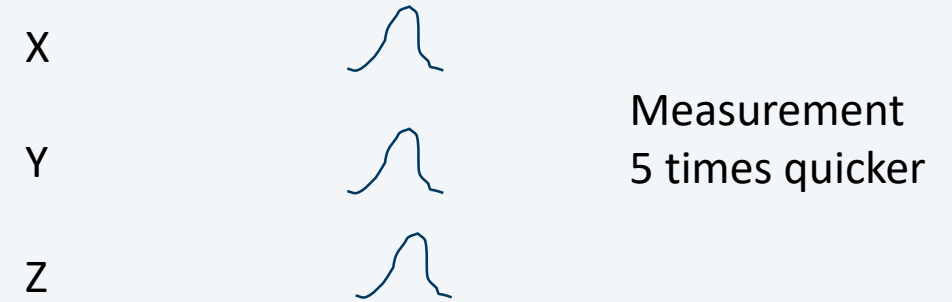
$$OSC = 0$$

Estimation possible from quick reference measurements

### Single shot

For one stress determination

Peaks of 1 unit time (for example)  
oscillations or not



Information available:

$$u(2\theta_{fitting})$$

$$OSC \geq 0$$

Information **not** available:

$$u(2\theta_{grain}) \text{ and } u(2\theta)$$

'Modelling'

# Multiple measurement

Very short  
neutron  
path length

$$u(2\theta) \approx (u(2\theta_{fitting})^2 + u(2\theta_{grain})^2)^{1/2}$$

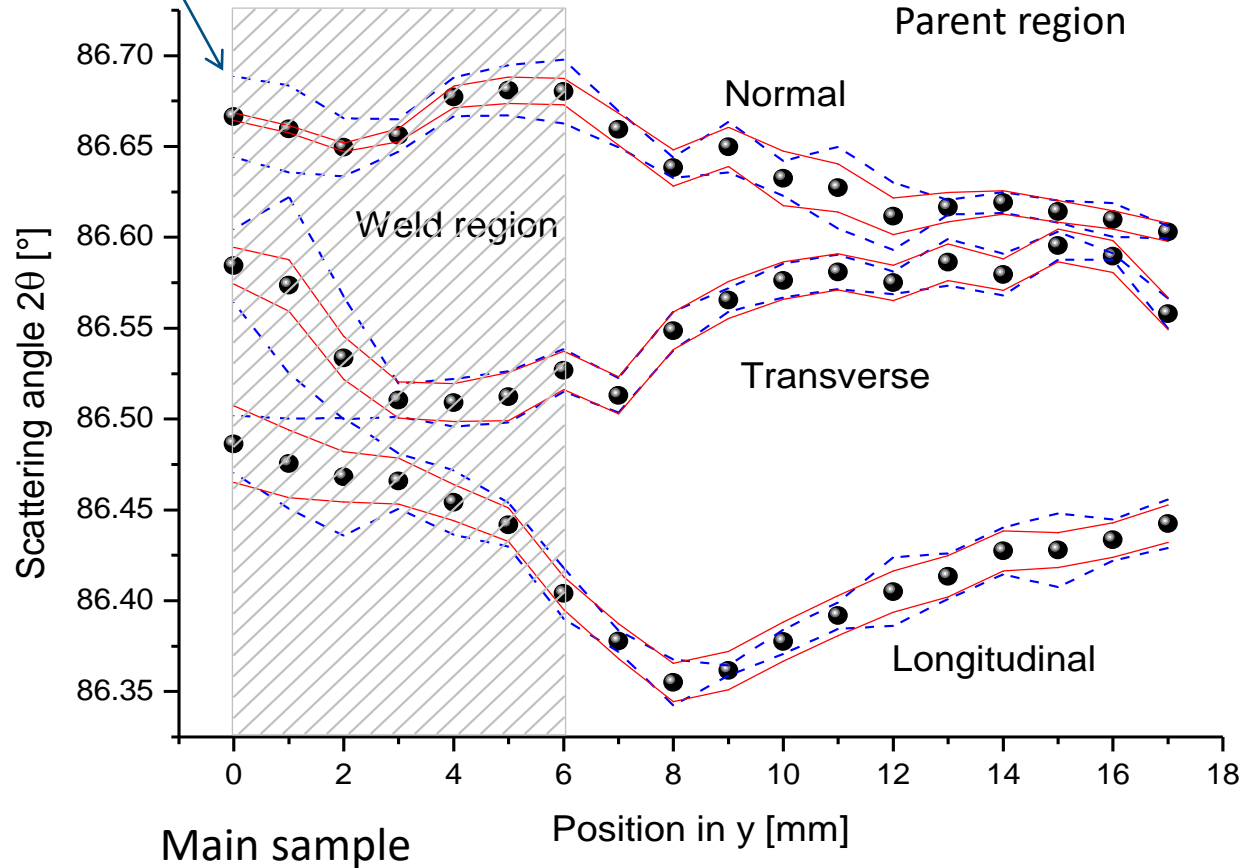
e.g. by measuring the specimen several times at slightly different rotation angles,  $-2^\circ$ ,  $-1^\circ$ ,  $0^\circ$ ,  $+1^\circ$ ,  $+2^\circ$  relative to the correct bisecting angle  $\omega$  and comparing the **average fit uncertainty values** with the **standard deviation of the  $2\theta$  values**.

The  $2\theta$  scattering angle along the line of measurements in the 3-pass slot weld in the three orthogonal directions, weld longitudinal, weld transverse and plate normal. The average  $2\theta$  values are shown here in black dots.

**The fitting uncertainty decreases with time, whereas the uncertainty due to grain size 'is fixed'.**

The contrast is seen most clearly in the normal direction near both surfaces ( $y = 0$  mm and 18 mm), where the short path length of the neutrons gives rise to a very strong diffraction signal in a short period of time. In the parent material (approximately starting from  $y = 6$  mm to 18 mm) the **red** and **blue** lines are approximately the same, suggesting that using only the fitting uncertainty is adequate in this region.

However this cannot be said of the weld region where the **red** and **blue** lines diverge, indicating a  $u(2\theta_{grain})$  contribution.



# Multiple and 'single shot' measurement

Top of weld region

Parent region

Relative rotation ( $\omega$ ) [°]	Top of weld region		Parent region	
	Transverse $y=2\text{mm}$ $2\theta$ [°]	Uncertainty of fit $u(2\theta_{\text{fitting}})$ [°]	Longitudinal $y=13\text{mm}$ $2\theta$ [°]	Uncertainty of fit $u(2\theta_{\text{fitting}})$ [°]
+2	86.516	0.009	86.413	0.011
+1	86.520	0.010	86.406	0.011
0	86.495	0.015	86.419	0.012
-1	86.577	0.015	86.398	0.011
-2	86.560	0.010	86.431	0.012
	Standard deviation $u(2\theta) = 0.034$ $u(2\theta_{\text{grain}}) = 0.032$	Average fitting uncertainty $u(2\theta_{\text{fitting}}) = 0.012$	Standard deviation $u(2\theta) = 0.013$ $u(2\theta_{\text{grain}}) = 0.005$	Average fitting uncertainty $u(2\theta_{\text{fitting}}) = 0.011$

$$u(2\theta) \approx (u(2\theta_{\text{fitting}})^2 + u(2\theta_{\text{grain}})^2)^{1/2}$$

$$u(2\theta_{\text{fitting}}) \propto \frac{1}{\sqrt{(\text{time})}} \quad u(2\theta_{\text{grain}}) \propto \frac{1}{\sqrt{(N_{DG})}} \quad \Rightarrow \quad u(2\theta) \propto \frac{1}{\sqrt{(\text{Number of measurements}^*)}}$$

\*Assuming the time of each measurement is the same and different grains are sampled and detected in each measurement



All uncertainties can be divided by  $\sqrt{5}$  in this case

# Multiple and 'single shot' measurement

$$u(2\theta_{fitting}) \propto \frac{1}{\sqrt{(time)}} \quad u(2\theta_{grain}) \propto \frac{1}{\sqrt{(N_{DG})}} \quad \Rightarrow \quad u(2\theta) \propto \frac{1}{\sqrt{(Number\ of\ measurements^*)}}$$

Top of weld region

**Transverse**  
**y= 2mm 2θ [°]**

Parent region

**Longitudinal**  
**y=13mm 2θ [°]**



$$u(2\theta) \approx (u(2\theta_{fitting})^2 + u(2\theta_{grain})^2)^{1/2}$$

$$0.034 \approx (0.012^2 + 0.032^2)^{1/2}$$

$$u(2\theta) \approx (u(2\theta_{fitting})^2 + u(2\theta_{grain})^2)^{1/2}$$

$$0.013 \approx (0.011^2 + 0.005^2)^{1/2}$$

**Single shot** Addition of 5 measurements of equal time and **sampling the same grains** (no oscillation or oscillation over the same grains)

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$$0.032 \approx (0.005^2 + 0.032^2)^{1/2}$$

$$0.007 \approx (0.005^2 + 0.005^2)^{1/2}$$

**Multiple** Addition of 5 Multiple measurements of equal time: **sampling different grains** (e.g. Stepwise oscillation)

**Multiple** Addition of 5 Multiple measurements of equal time: **sampling different grains** (e.g. Stepwise oscillation)

$$0.015 \approx (0.005^2 + 0.014^2)^{1/2}$$

$$0.005 \approx (0.005^2 + 0.002^2)^{1/2}$$

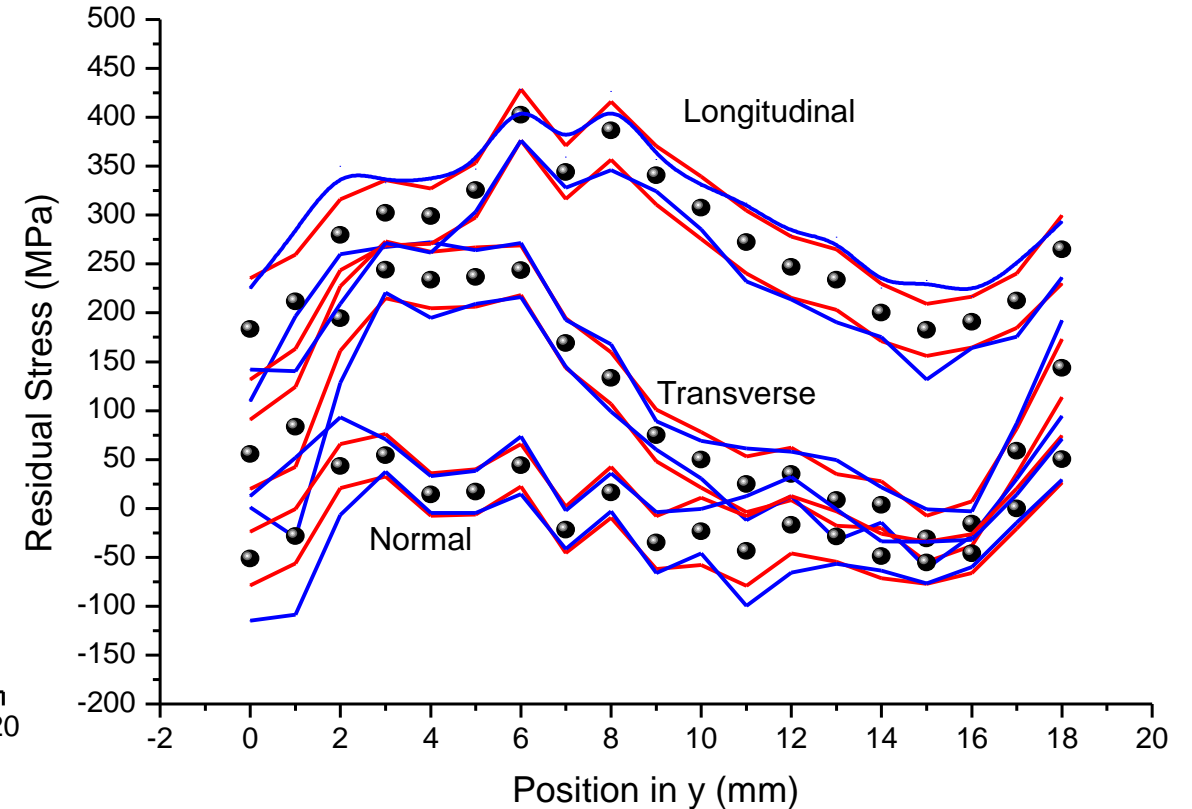
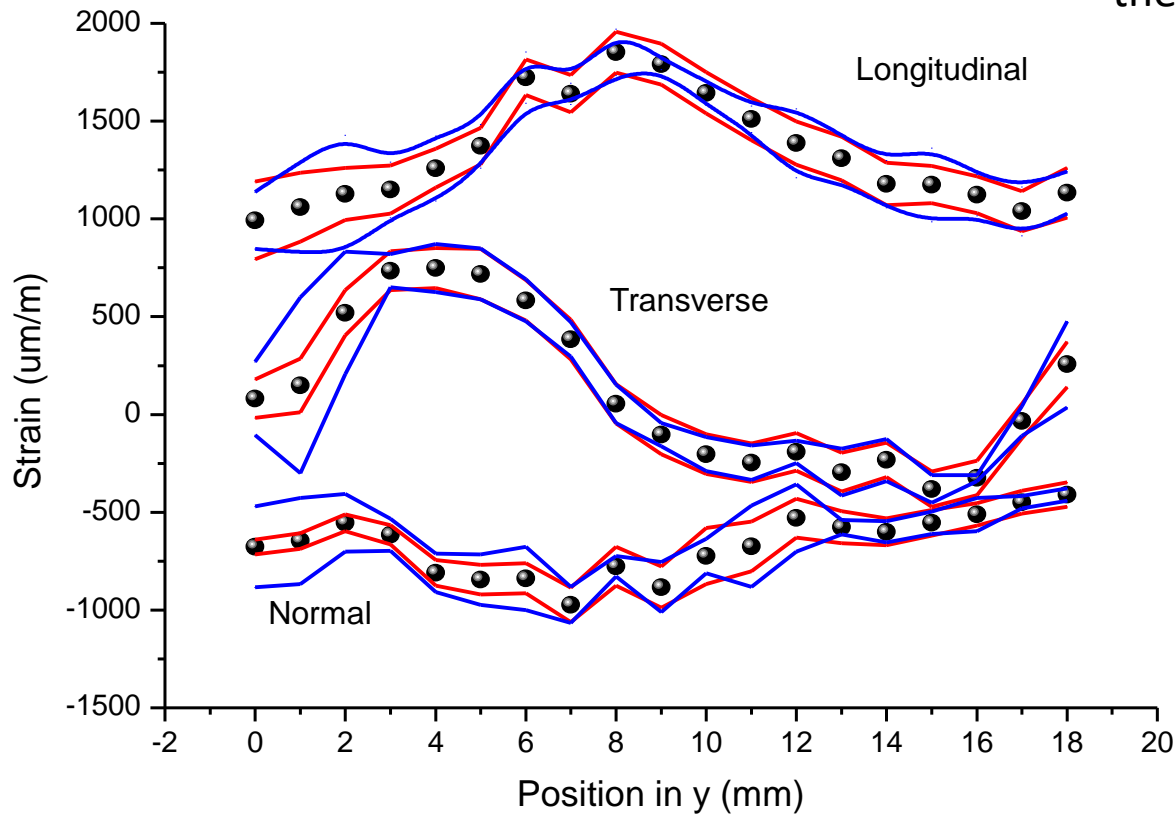
# Propagation of the uncertainty in 2 theta

$$u(2\theta) \longrightarrow u(\varepsilon) \longrightarrow u(\sigma)$$

Measured BD line **5 times**. Instead of oscillating specimen, made 5 scans in -2, -1, 0, 1, 2 degrees offset in omega. **For each data set we are effectively looking at different sets of grains.** Looked at results individually. Looked at summed results.

$$u(2\theta) \approx (u(2\theta_{\text{fitting}})^2 + u(2\theta_{\text{grain}})^2)^{1/2}$$

The uncertainty values shown represent the values we should get if we had only measured once. Because we have **measured 5 times**, the uncertainties will actually be **SQRT(5) times less**.



# Multiple measurement and 'single shot' measurement

Need a more realistic value

$$u(\sigma_{xx}) \cong \left[ \left( \frac{E_{hkl}(1 - \nu_{hkl})}{(1 + \nu_{hkl})(1 - 2\nu_{hkl})} u(\varepsilon_{xx}) \right)^2 + \left( \frac{\nu_{hkl} E_{hkl}}{(1 + \nu_{hkl})(1 - 2\nu_{hkl})} \left( u(\varepsilon_{yy})^2 + u(\varepsilon_{zz})^2 \right) \right)^2 \right]^{1/2}$$

$$u(\varepsilon) = \frac{1}{\tan\theta_0} \left[ u(\theta_{fitting})^2 + u(\theta_{grain})^2 + u(\theta_{0-fit})^2 + u(\theta_{0-grain})^2 \right]^{1/2}$$

$$u(2\theta) \approx \left( u(2\theta_{fitting})^2 + u(2\theta_{grain})^2 \right)^{1/2}$$

Multiple measurement: time consuming

'Main Sample'



$$u(2\theta_{fitting})$$



Check if this is accurate in the first place.

$$u(2\theta_{grain})$$



See if we can 'model' this from material properties and instrumental parameters for single shot measurements.

$$u(2\theta_0) \approx \left( u(2\theta_{0-fit})^2 + u(2\theta_{0-grain})^2 \right)^{1/2}$$

Multiple measurement: quick

'Reference sample'





# Counting statistics versus grain size statistics

Equation from:  
Withers, P. J.,  
Daymond, M. R. &  
Johnson, M. W. (2001).  
*J. Appl. Crystallogr.* **34**,  
737-743.

$$u(2\theta) \approx (u(2\theta_{fitting})^2 + u(2\theta_{grain})^2)^{1/2}$$

$$u(2\theta_{fitting})^2 \approx \left(\frac{SD_{Gauss}^2}{I}\right) \left[1 + 2\left(2^{\frac{1}{2}}\right)\frac{B}{H}\right]$$

Time Dependent  $u(2\theta_{fitting}) \propto \frac{1}{\sqrt{(time)}}$

$$u(2\theta_{grain}) \approx \frac{0.5 * SD_{Gauss}}{(N_{DG})^{1/2}}$$

Not Time Dependent  $u(2\theta_{grain}) \propto \frac{1}{\sqrt{(N_{DG})}}$

'Wimporry model'

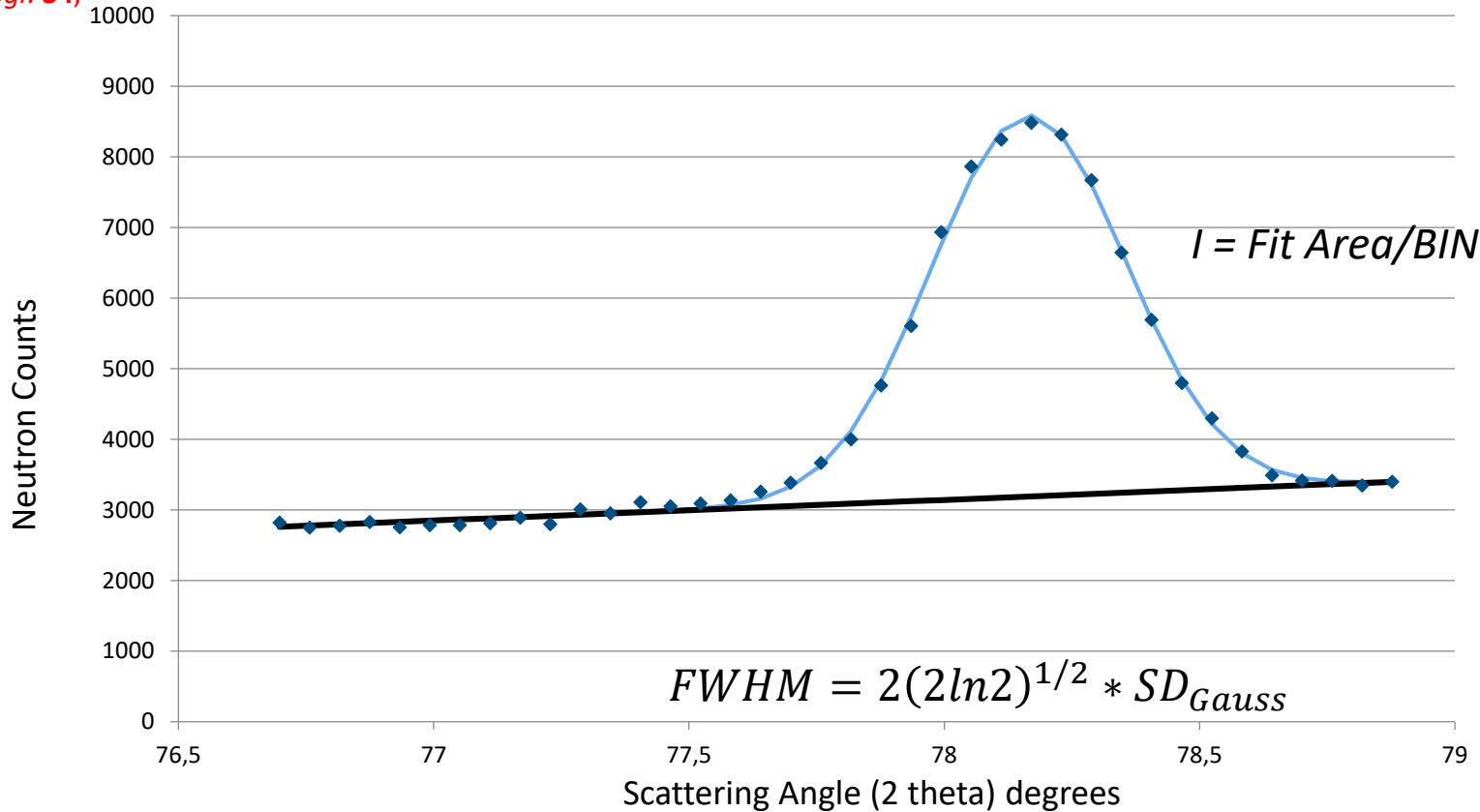
Parameter	Description
$SD_{Gauss}$	$FWHM = 2(2\ln 2)^{1/2} * SD_{Gauss}$
$I$	Integrated intensity (area under Gaussian or Voigt divided by Bin Size)
$B$	Background value at centre of peak
$H$	Height of Gaussian or Voigt Peak
$N_{DG}$	Number of <u>detected</u> diffracting grains

# Is the returned fitting uncertainty correct?

2th(°)	2th err (°)	Fit Area	FWHM (°)	SDGauss (°)	H	B	BIN (°)	I
78.1650	0.0018	2583	0.450	0.191	5394	3190	0.0589	43854

Equation from:  
Withers, P. J.,  
Daymond, M. R. &  
Johnson, M. W. (2001).  
*J. Appl. Crystallogr.* **34**,  
737-743.

$$u(2\theta_{fitting})^2 \approx \left( \frac{SD_{Gauss}^2}{I} \right) \left[ 1 + 2 \left( 2^{\frac{1}{2}} \right) \frac{B}{H} \right] \quad u(2\theta_{fitting}) = 0.0015^\circ$$



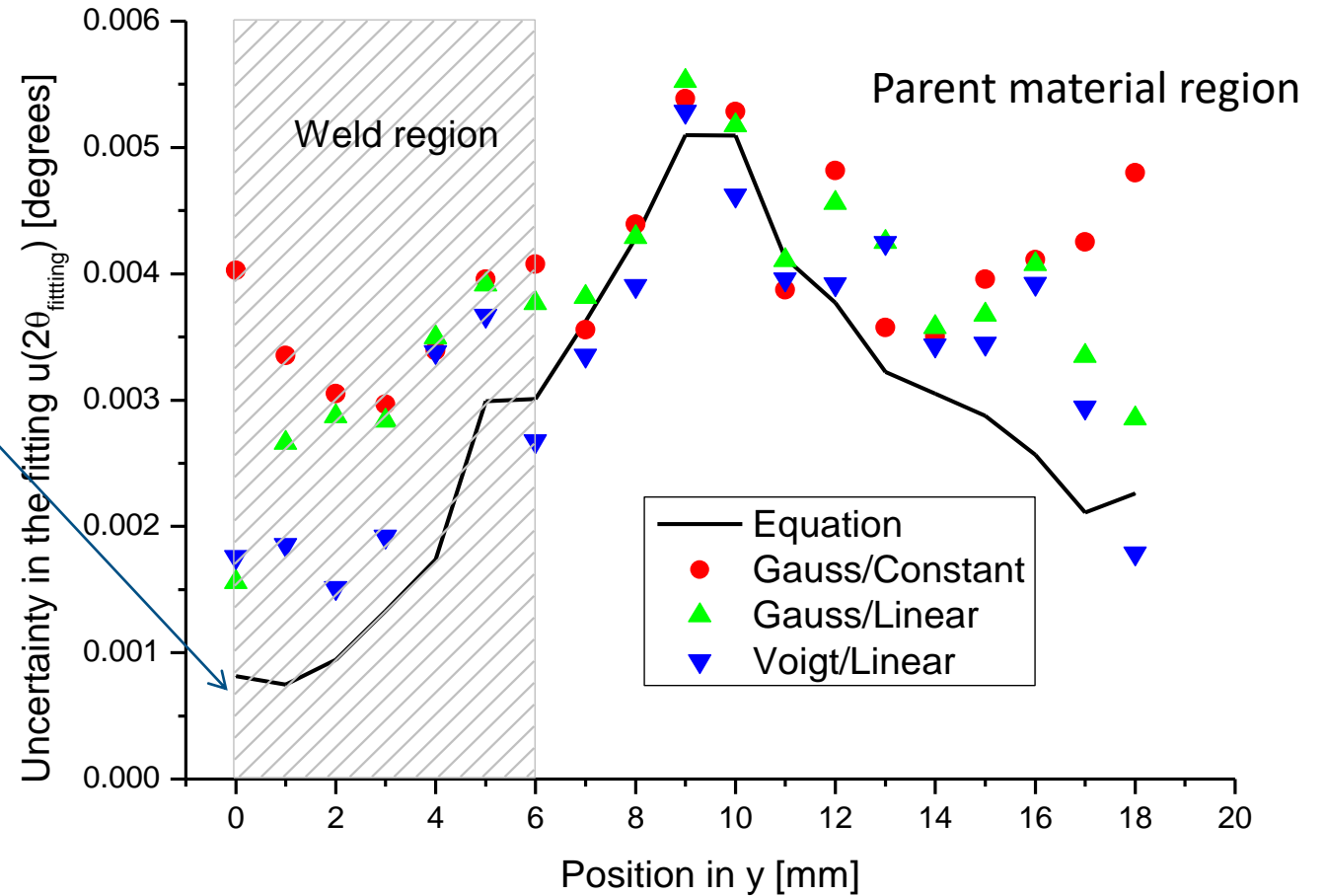
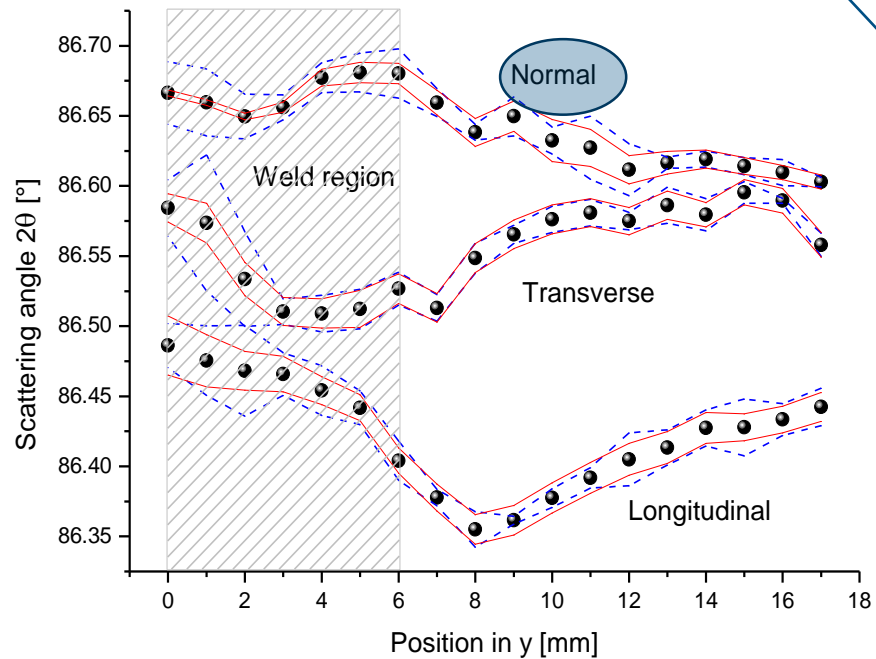
*Geometrical area*  
Assuming a triangle  
=  $H * 0.5$   
=  $5394 * 0.5$   
= 2697

*Gaussian fit area = 2583*

Number of neutrons  
(Integrated Intensity)  
 $I = \text{Fit Area} / \text{BIN}$   
=  $2583 / 0.0589 = 43854$

# Checking the fit uncertainty with the equation

$$(2\theta_{fitting})^2 \approx \left( \frac{SD_{Gauss}^2}{I} \right) \left[ 1 + 2 \left( 2^{\frac{1}{2}} \right) \frac{B}{H} \right]$$



The returned fitting uncertainty values from the fit program are closer to the equation values in this order:

**Gaussian/Constant background** =worst, **Gaussian/Linear background** = better, **Voigt/Linear background** =best

T. Gnaeupel-Herold, H. J. Prask, R. J. Fields, T. J. Foecke, Z. C. Xia, U. Lienert, A synchrotron study of residual stresses in a Al6022 deep drawn cup, Mater. Sci. Eng., A 366 (2004) 104–113.

'Wimpory Model'

$$u(2\theta_{grain}) \approx \frac{0.5 * SD_{Gauss}}{(N_{DG})^{1/2}}$$

From multiple measurement

$$N_{DG} \approx \left(\frac{I}{u(I)}\right)^2$$

$$N_{DG} \approx \left(P * \frac{gv}{(S_G)^3}\right)$$

From instrument and material parameters (single shot)

$$P \approx \frac{(D_H + \eta_M) * (OSC + \omega_M)}{4\pi} * m^{hkl}$$

Radians {

Symbol	Parameter	How to increase N <sub>DG</sub>
gv	Gauge Volume	Increase
S <sub>G</sub>	Grain size	Decrease
D <sub>H</sub>	Angular detector height	Increase
η <sub>M</sub>	Grain mosaicity in azimuthal direction, i.e. along the diffraction ring	Increase
OSC	The total angular oscillation of the sample around the ω-axis	Increase
ω <sub>M</sub>	Grain mosaicity around the ω-axis	Increase
m <sup>hkl</sup>	multiplicity of the particular Bragg reflection	Increase

# Uncertainty versus Time and $N_{DG}$

$$u(2\theta_0) \approx (u(2\theta_{0-fitting})^2 + u(2\theta_{0-grain})^2)^{1/2}$$

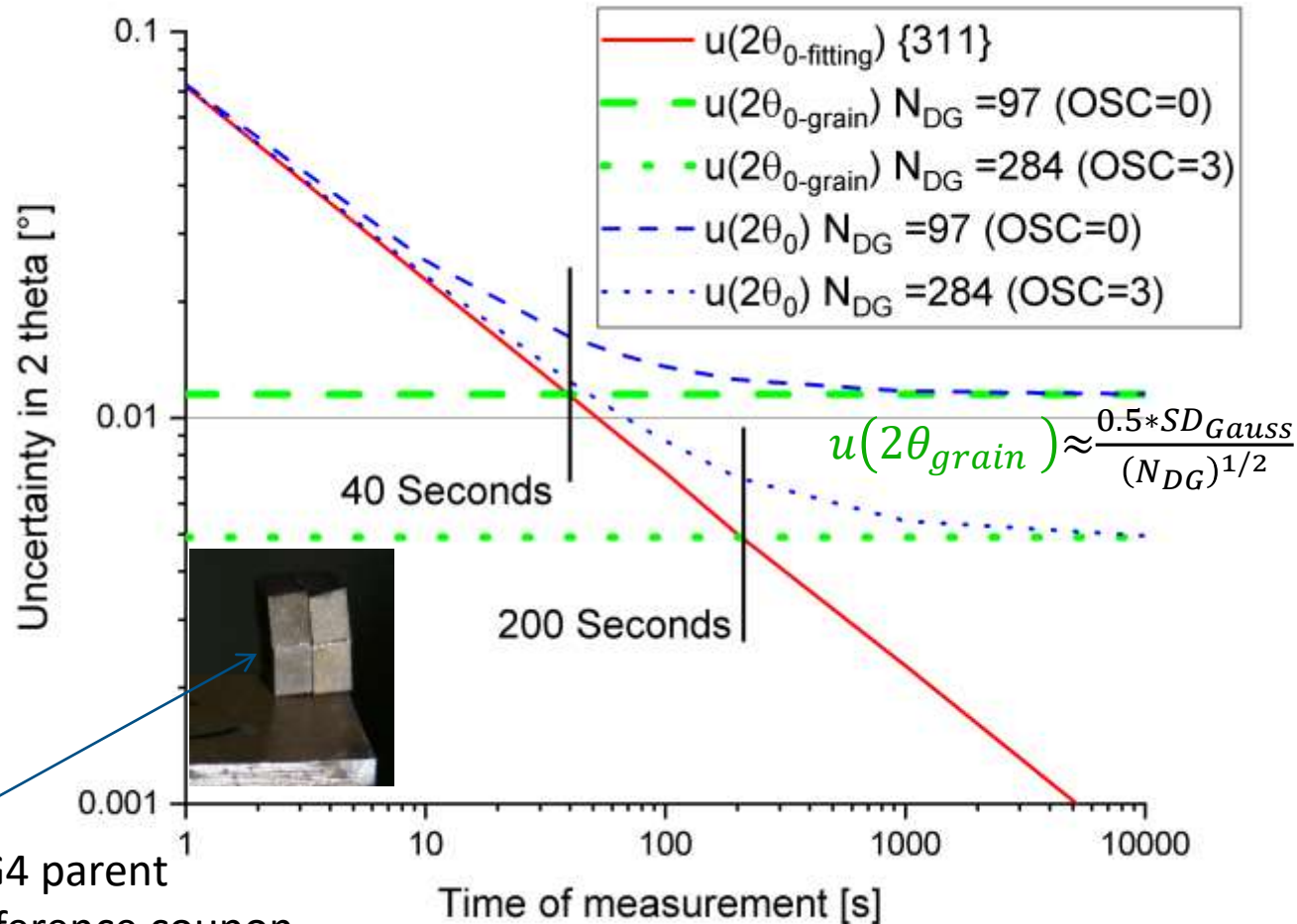
This shows the fitting uncertainty as a function of time for the {3 1 1} reflection for the 'black' TG4 parent reference coupon on E3.

The grain size uncertainty for OSC=0 and OSC=3 are also shown.

For this particular experimental set-up and specimen, it takes 100 seconds to get to a fitting uncertainty of about  $\pm 0.01^\circ$ .

If one does not oscillate (OSC=0) the total uncertainty cannot get any lower as one has a value of  $N_{DG} \approx 97$  which places an upper bound of the value of  $u(2\theta_{0-grain}) \approx \pm 0.0115^\circ$ .

It takes 1000 seconds to get to a fitting uncertainty of about  $\pm 0.005^\circ$ . If one oscillates (OSC=3) the total uncertainty cannot get any lower as one has a value of  $N_{DG} \approx 284$  which places an upper bound of the value of  $u(2\theta_{0-grain}) \approx \pm 0.0049^\circ$ .



TG4 parent  
reference coupon  
on E3

# Testing the model with the reference samples

From multiple  
measurement

$$N_{DG} \approx \left( \frac{I}{u(I)} \right)^2$$

$$u(2\theta_{grain}) \approx \frac{0.5 * SD_{Gauss}}{(N_{DG})^{1/2}}$$

$$N_{DG} \approx \left( P * \frac{gv}{(S_G)^3} \right)$$

$$P \approx \frac{(D_H + \eta_M) * (OSC + \omega_M)}{4\pi} * m^{hkl}$$

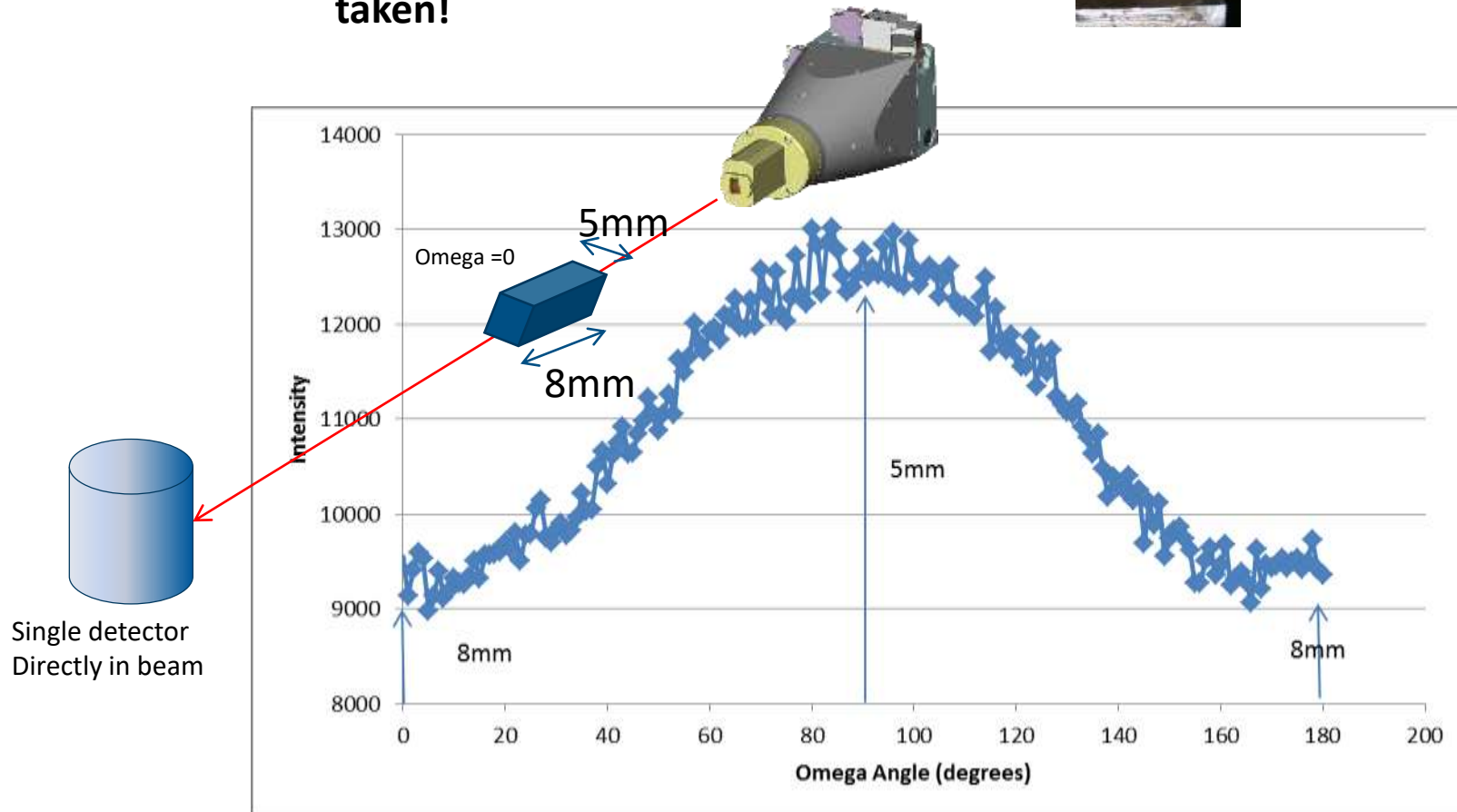


Experiment	Expectation
Increase $gv$ (constant $P$ )	$S_G$ Value remains constant, $N_{DG}$ value increases
Increase $P$ ( $m^{hkl}$ ) (constant $gv$ )	$S_G$ Value remains constant, $N_{DG}$ value increases
Increase $P$ ( $OSC$ ) (constant $gv$ )	$S_G$ Value remains constant, $N_{DG}$ value increases
$u(2\theta_{grain})$ versus $SD_{Gauss}$	Should be a straight line going through zero

# Estimating the number of diffracting grains from measurement

$$N_{DG} \approx \left( \frac{I}{u(I)} \right)^2$$

From multiple measurement  
**Care has to be taken!**



If the sample is **not round**, the single detector in the primary neutron beam can be used to indicate thickness for the normalization of the intensity data.

This was one of the many advantages of measuring on E3

Increase gv  
Increase P ( $m^{hkl}$ )

$S_G$  Value remains constant,  $N_{DG}$  value increases  
 $S_G$  Value remains constant,  $N_{DG}$  value increases



Parent

	Ave $2\theta_0$	$SD_{Gauss}$ [°]	B/H	Fitted steps	Ave I	$u(I)$	$N_{DG}$	$u(2\theta_0)$ [°]	$u(2\theta_0-fitting)$ [°]	$u(2\theta_0-grain)$ [°]	gv [mm <sup>3</sup> ]	$S_G$ [μm]
1	86.616	0.178	1.02	11	14346	2120	46	0.0068	0.0029	0.0062	2	42
2	86.613	0.183	0.33	28	13343	1253	113	0.0094	0.0022	0.0091	8	84
3	86.616	0.187	0.18	28	13900	1182	138	0.0072	0.0019	0.0069	18	91
4	86.613	0.192	0.12	28	13795	1007	188	0.0073	0.0019	0.0070	32	109
1	86.635	0.213	0.22	55	12907	3746	11.9	0.0148	0.0024	0.0146	18	137
2	86.629	0.215	0.15	55	13201	3933	11.3	0.0141	0.0022	0.0139	32	159
1	86.647	0.197	0.40	74	11451	8664	1.7	0.0432	0.0027	0.0431	8	227
2	86.637	0.199	0.18	109	14719	10245	2.1	0.0280	0.0020	0.0280	18	221
3	86.637	0.204	0.13	55	13821	9931	1.9	0.0301	0.0020	0.0301	32	276

Weld

Top/Bottom

$$hkl \{3 \ 1 \ 1\} m^{hkl} = 24$$

Results of the  $hkl \{3 \ 1 \ 1\}$  Bragg reflection ( $P=0.01131$ ) for TG4 reference coupons SET Y.

$$N_{DG} \approx \left( \frac{I}{u(I)} \right)^2$$

Parent

	Ave $2\theta_0$	$SD_{Gauss}$ [°]	B/H	Fitted steps	Ave I	$u(I)$	$N_{DG}$	$u(2\theta_0)$ [°]	$u(2\theta_0-fitting)$ [°]	$u(2\theta_0-grain)$ [°]	gv [mm <sup>3</sup> ]	$S_G$ [μm]
1	91.645	0.209	2.98	11	5462	1447	14.3	0.0281	0.0087	0.0267	2	69
2	91.639	0.213	1.00	28	4706	965	23.8	0.0133	0.0061	0.0119	8	63
3	91.637	0.215	0.54	28	4782	882	29.4	0.0116	0.0049	0.0105	18	76
4	91.635	0.220	0.38	28	4681	691	45.9	0.0121	0.0046	0.0112	32	94
1	91.658	0.227	0.47	51	6281	3861	2.6	0.0360	0.0044	0.0357	18	165
2	91.659	0.230	0.33	55	5584	3839	2.1	0.0296	0.0043	0.0293	32	174
1	91.664	0.232	1.22	42	7399	7276	1.0	0.0603	0.0057	0.0600	8	175
2	91.660	0.234	0.87	62	5494	5625	1.0	0.0531	0.0059	0.0528	18	210
3	91.657	0.234	0.62	40	3961	4644	0.7	0.0571	0.0062	0.0567	32	267

Weld

Top/Bottom

$$hkl \{2 \ 2 \ 2\} m^{hkl} = 8$$

Results of the  $hkl \{2 \ 2 \ 2\}$  Bragg reflection ( $P=0.00377$ ) for TG4 reference coupons SET Y.

$$S_G \approx \left( \frac{P \cdot gv}{\left( \frac{0.5 \cdot SD_{Gauss}}{u(2\theta_{grain})} \right)^2} \right)^{1/3}$$



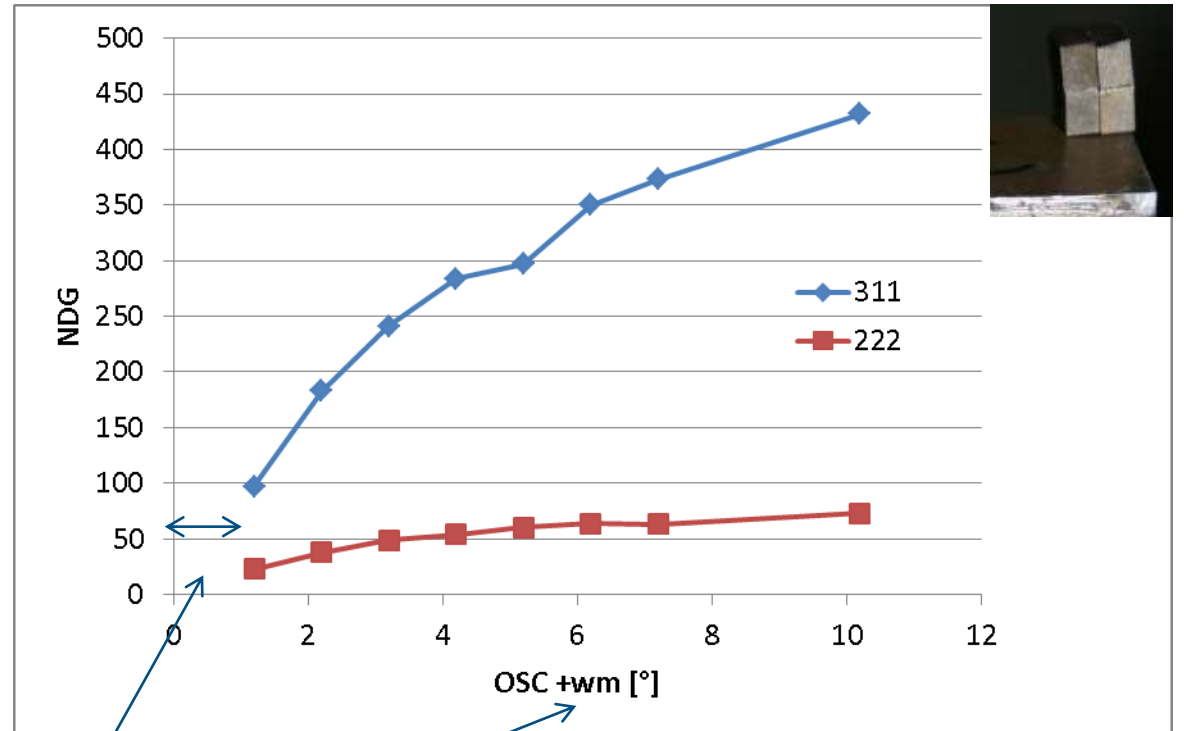
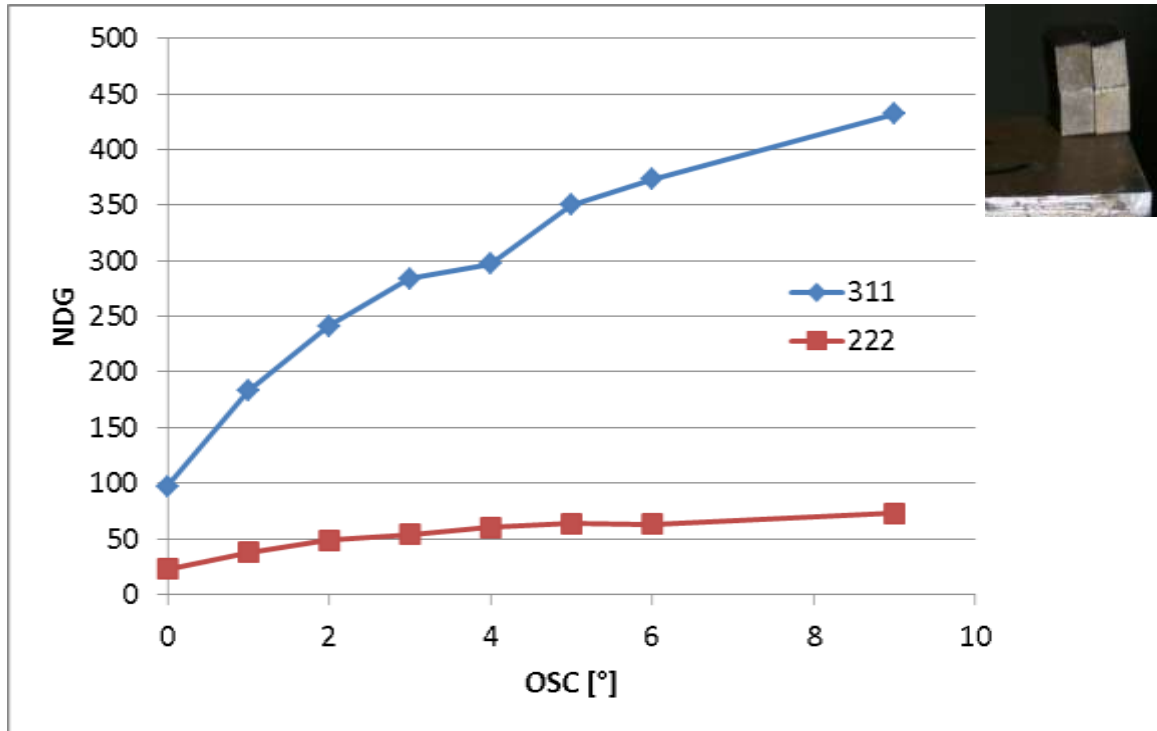
# Mosaicity

Assumed to be the same

Increase P (OSC)  
 S<sub>G</sub> Value remains constant, N<sub>DG</sub> value increases

$$N_{DG} \approx \left(\frac{I}{u(I)}\right)^2$$

$$P \approx \frac{(D_H + \eta_M) * (OSC + \omega_M)}{4\pi} * m^{hkl}$$



For the analysis we used an estimated value of  $\omega_m \approx 1.2^\circ$

Measurements made at the ESRF by the JRC saw a range of mosaicity values of up to  $1.55^\circ$  for the {1 1 1} reflection.

# Stepwise Oscillation of the TG4 'black' parent reference coupon 311

$$N_{DG} \approx \left(\frac{I}{u(I)}\right)^2$$

$$u(2\theta_{0-fitting})^2 \approx \left(\frac{SD_{Gauss}^2}{I}\right) \left[1 + 2\left(2\frac{1}{2}\right)\frac{B}{H}\right]$$

$$P \approx \frac{(D_H + \eta_M) * (OSC + \omega_M)}{4\pi} * m^{hkl}$$

$$S_G \approx \left(\frac{P * gv}{\left(\frac{0.5 * SD_{Gauss}}{u(2\theta_{grain})}\right)^2}\right)^{1/3}$$

$$u(2\theta_0) \approx (u(2\theta_{0-fitting})^2 + u(2\theta_{0-grain})^2)^{1/2}$$

$$u(2\theta_{grain-0}) \approx \frac{0.5 * SD_{Gauss}}{(N_{DG})^{1/2}}$$



row	N <sub>DG</sub>	No. of peaks	OSC [°]	Average 2θ <sub>0</sub> [°]	u(2θ <sub>0</sub> ) [°]	u(2θ <sub>0-fitting</sub> ) [°]	u(2θ <sub>0-grain</sub> ) (a) [°]	u(2θ <sub>0-grain</sub> ) (b) [°]	P	S <sub>G</sub> [μm]
1	97	219	0	85.3688	0.0120	0.0037	0.0115	0.0111	0.0115	100
2	183	109	1	85.3688	0.0077	0.0026	0.0073	0.0081	0.0210	91
3	241	73	2	85.3688	0.0065	0.0021	0.0061	0.0070	0.0306	92
4	284	54	3	85.3687	0.0053	0.0018	0.0049	0.0065	0.0401	87
5	297	43	4	85.3687	0.0048	0.0016	0.0045	0.0063	0.0497	87
6	350	36	5	85.3687	0.0050	0.0015	0.0048	0.0058	0.0592	97
7	373	31	6	85.3687	0.0045	0.0014	0.0043	0.0057	0.0687	95
10	432	21	9	85.3688	0.0035	0.0011	0.0033	0.0053	0.0974	89

Increase P (OSC)

S<sub>G</sub> Value remains constant, N<sub>DG</sub> value increases

Average FWHM of peaks = 0.52°

# Stepwise Oscillation of the TG4 'black' parent reference coupon 222

$$N_{DG} \approx \left(\frac{I}{u(I)}\right)^2$$

$$u(2\theta_{0-fitting})^2 \approx \left(\frac{SD_{Gauss}^2}{I}\right) \left[1 + 2\left(2\frac{1}{2}\right)\frac{B}{H}\right]$$

$$P \approx \frac{(D_H + \eta_M) * (OSC + \omega_M)}{4\pi} * m^{hkl}$$

$$S_G \approx \left(\frac{P * gv}{\left(\frac{0.5 * SD_{Gauss}}{u(2\theta_{grain})}\right)^2}\right)^{1/3}$$

$$u(2\theta_0) \approx (u(2\theta_{0-fitting})^2 + u(2\theta_{0-grain})^2)^{1/2}$$

$$u(2\theta_{grain-0}) \approx \frac{0.5 * SD_{Gauss}}{(N_{DG})^{1/2}}$$



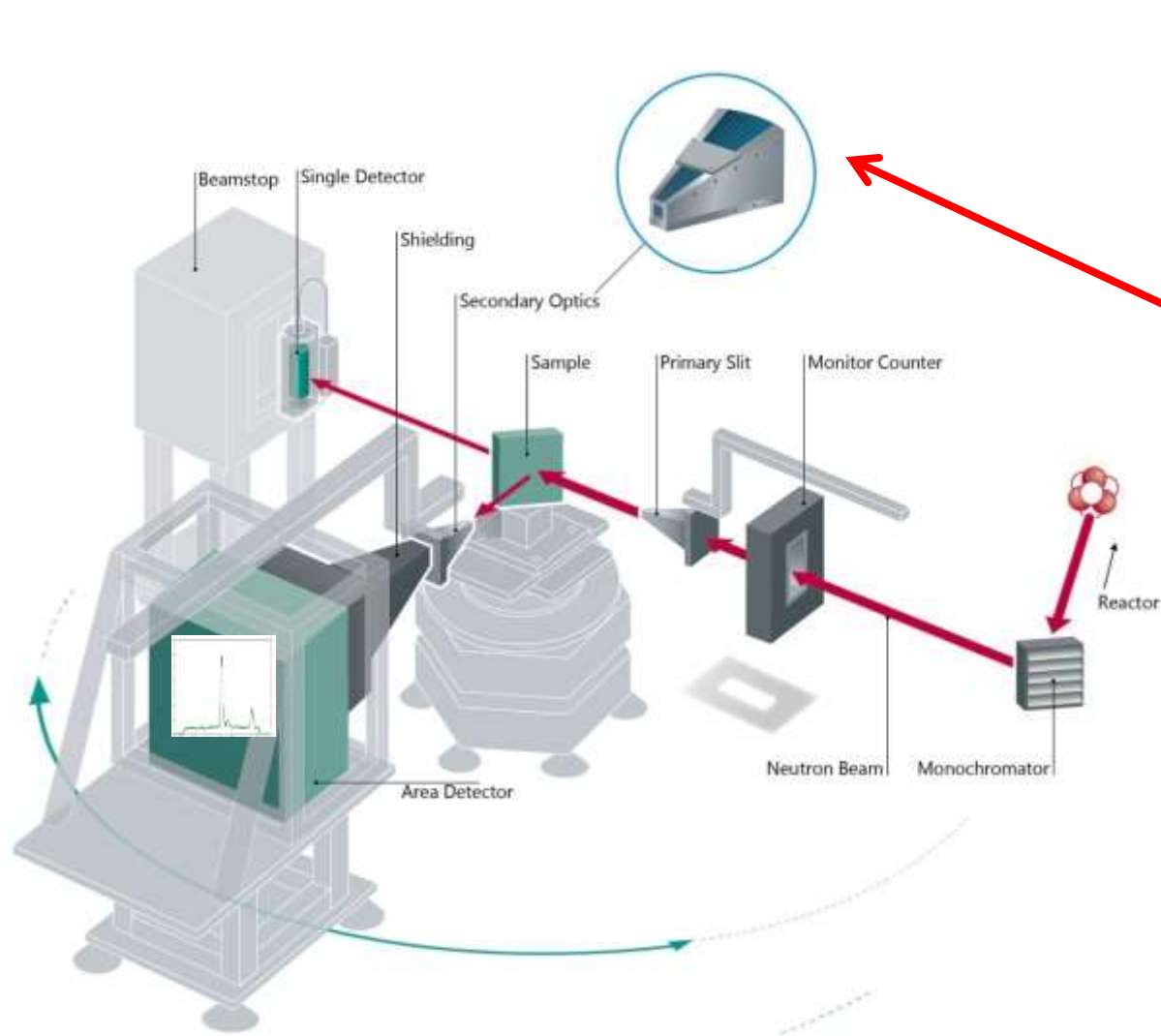
row	N <sub>DG</sub>	No. of peaks	OSC [°]	Average 2θ <sub>0</sub> [°]	u(2θ <sub>0</sub> ) [°]	u(2θ <sub>0-fitting</sub> ) [°]	u(2θ <sub>0-grain</sub> ) (a) [°]	u(2θ <sub>0-grain</sub> ) (b) [°]	P	S <sub>G</sub> [μm]
1	23	219	0	90.1512	0.0260	0.0113	0.0235	0.0249	0.0038	106
2	38	109	1	90.1512	0.0176	0.0080	0.0157	0.0194	0.0070	99
3	49	73	2	90.1512	0.0153	0.0065	0.0138	0.0170	0.0102	103
4	54	54	3	90.1513	0.0136	0.0056	0.0124	0.0161	0.0134	105
5	60	43	4	90.1514	0.0135	0.0050	0.0125	0.0154	0.0166	114
6	64	36	5	90.1513	0.0124	0.0046	0.0115	0.0148	0.0197	114
7	63	31	6	90.1512	0.0122	0.0043	0.0114	0.0149	0.0229	119
10	73	21	9	90.1514	0.0112	0.0035	0.0106	0.0139	0.0325	128

Increase P (OSC)

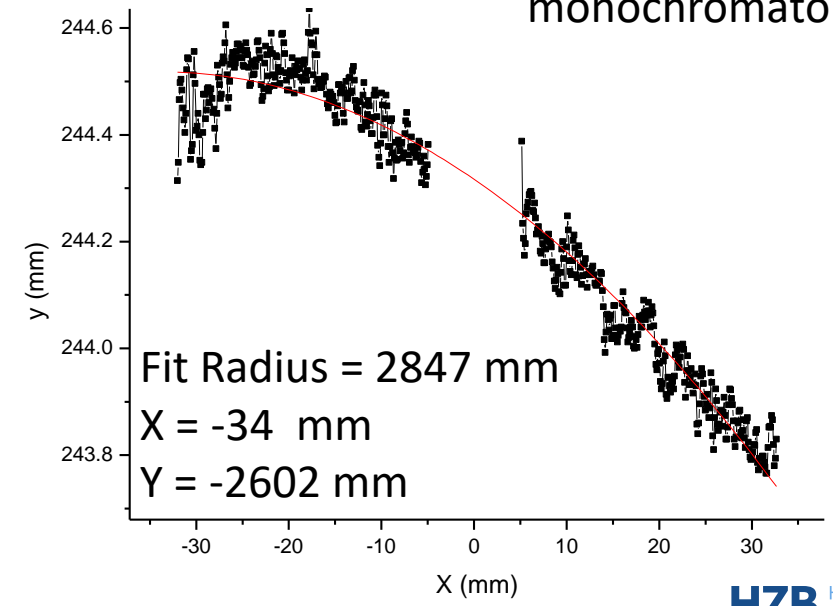
S<sub>G</sub> Value remains constant, N<sub>DG</sub> value increases

Average FWHM of peaks = 0.57°

$u(2\theta_{grain})$  versus  $SD_{Gauss}$  Should be a straight line going through zero



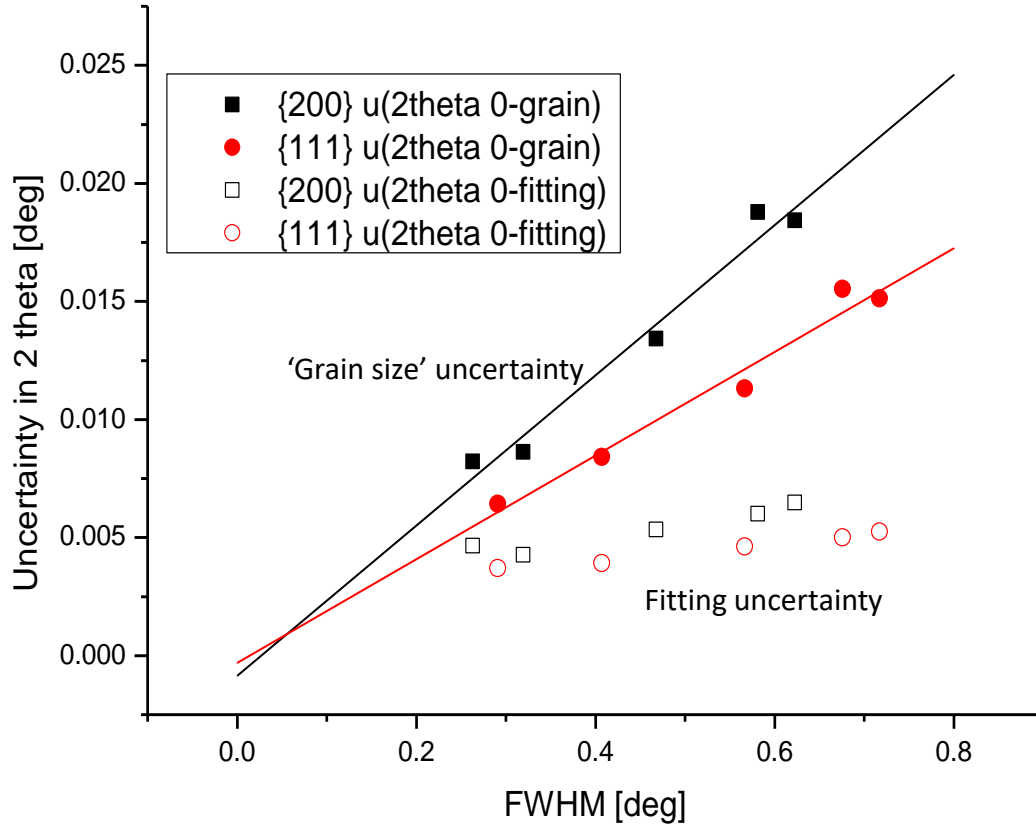
Bending radius of monochromator



$u(2\theta_{grain})$  versus  $SD_{Gauss}$  Should be a straight line going through zero



Grain size uncertainty contribution as a function of instrument resolution compared to corresponding fitting uncertainties for this particular data set.



$$u(2\theta_{grain}) \approx \frac{0.5 * SD_{Gauss}}{(N_{DG})^{1/2}} = \frac{0.21 * FWHM}{(N_{DG})^{1/2}}$$

$$N_{DG} \approx \left( \frac{I}{u(I)} \right)^2$$

Adjusted for path length

<i>hkl</i>	$2\theta_{hkl}$	$m^{hkl}$	$N_{DG}$	$N_{DG}$
200	48.6°	6	45 ± 10	69 ± 14
111	41.8°	8	94 ± 20	121 ± 6
111/200		1.33	2.10 ± 0.91	1.75 ± 0.44

Greater proportion of Debye-Scherrer ring on detector

# Verification of the model

Experiment	Expectation	Result
Increase $gv$	$S_G$ Value remains constant, $N_{DG}$ value increases	Verified
Increase $P$ ( $m^{hkl}$ )	$S_G$ Value remains constant, $N_{DG}$ value increases	Verified
Increase $P$ ( $OSC$ )	$S_G$ Value remains constant, $N_{DG}$ value increases	Verified
$u(2\theta_{grain})$ versus $SD_{Gauss}$	Should be a straight line going through zero	Verified

$$u(2\theta_{grain}) \approx \frac{0.5 * SD_{Gauss}}{(N_{DG})^{1/2}} \approx \frac{0.5 * SD_{Gauss}}{\left(P * \frac{gv}{(S_G)^3}\right)^{1/2}}$$

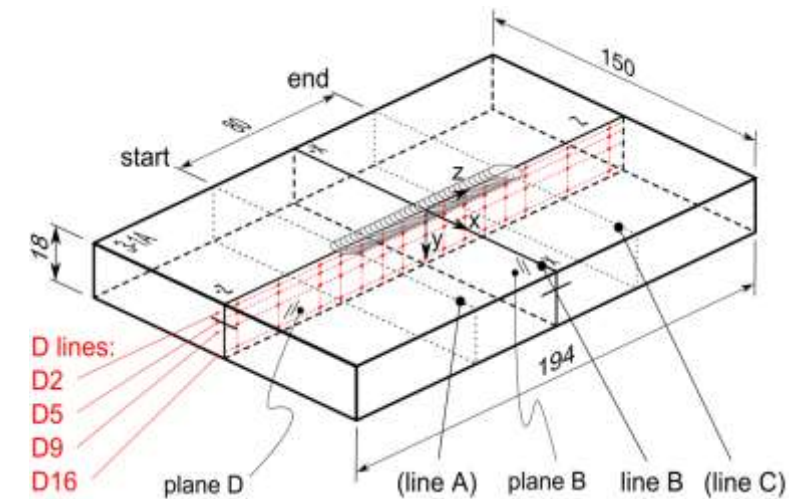
$$S_G \approx \left( \frac{P * gv}{\left(\frac{0.5 * SD_{Gauss}}{u(2\theta_{grain})}\right)^2} \right)^{1/3}$$

# Applying the model to the main sample

The D lines were 'single shot'

The D9 line is completely within the parent material of the specimen: Should show up the least grain size effect

Position	Reference	Grain size estimation $S_G$	Main sample
Parent	Black	90 $\mu$ m	D9
Weld Bottom	Green	180 $\mu$ m	D5
Weld Top	Red	260 $\mu$ m	D2



$$P \approx \frac{(D_H + \eta_M) * (OSC + \omega_M)}{4\pi} * m^{hkl}$$

$$N_{DG} \approx \left( P * \frac{gv}{(S_G)^3} \right)$$

$$u(2\theta_{grain}) \approx \frac{0.5 * SD_{Gauss}}{(N_{DG})^{1/2}}$$

## Applying the model to main sample

Position	Reference	Grain size estimation $S_G$	Main sample	$hkl$	$E_{hkl}$ [GPa]	$\nu_{hkl}$
Parent	Black	90 $\mu\text{m}$	D9	311	183.6	0.306
Weld Bottom	Green	180 $\mu\text{m}$	D5	222	209.4	0.278
Weld Top	Red	260 $\mu\text{m}$	D2			

$$u(\varepsilon) = \frac{1}{\tan\theta_0} \left[ u(\theta_{fitting})^2 + u(\theta_{grain})^2 + u(\theta_{0-fit})^2 + u(\theta_{0-grain})^2 \right]^{\frac{1}{2}}$$

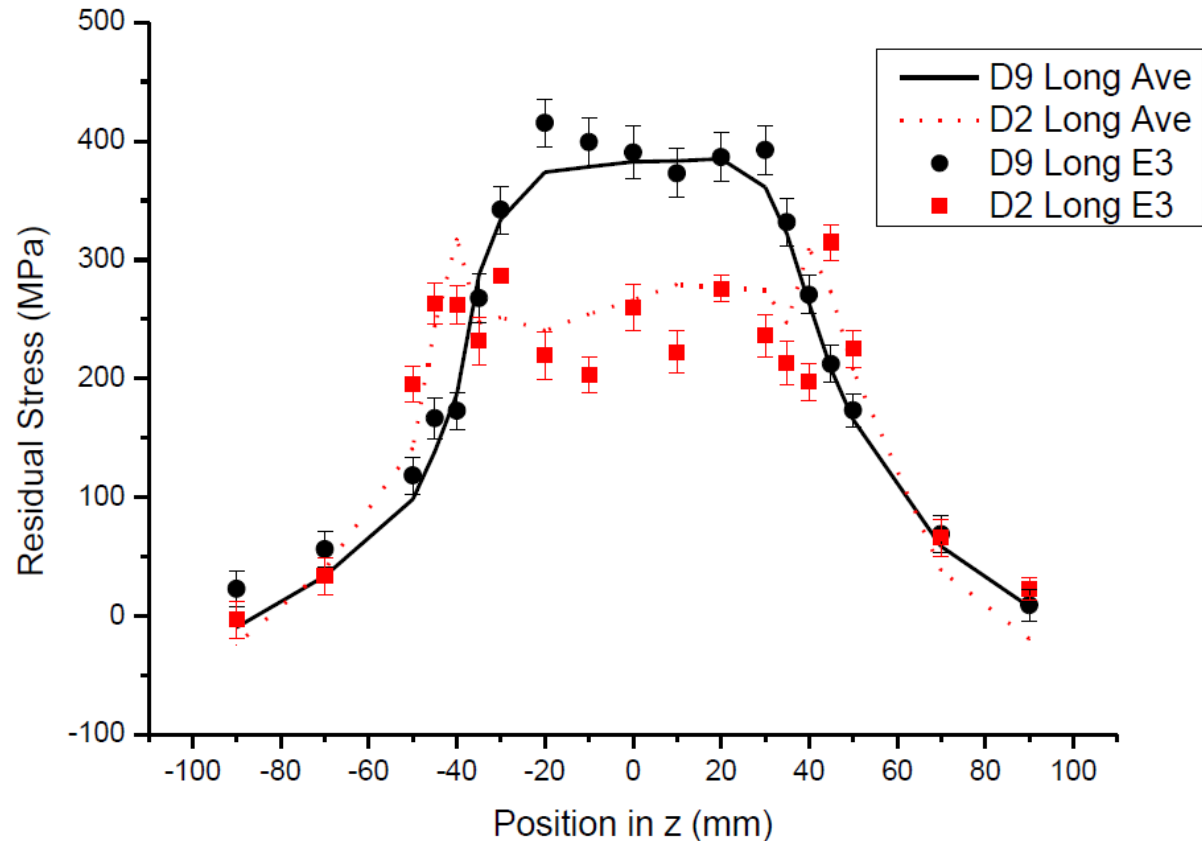
$u(2\theta)$        $u(2\theta_0)$

$$u(\sigma_{xx}) \cong \left[ \left( \frac{E_{hkl}(1 - \nu_{hkl})}{(1 + \nu_{hkl})(1 - 2\nu_{hkl})} u(\varepsilon_{xx}) \right)^2 + \left( \frac{\nu_{hkl} E_{hkl}}{(1 + \nu_{hkl})(1 - 2\nu_{hkl})} \right)^2 (u(\varepsilon_{yy})^2 + u(\varepsilon_{zz})^2) \right]^{\frac{1}{2}}$$

(Assuming  $u(\varepsilon_{xx}) = u(\varepsilon_{yy}) = u(\varepsilon_{zz})$ )



# 'Actual Uncertainties' Comparison to Robust Average



$u(\sigma)$

For the D2 line the measurement points between  $z = -40$  and  $40$  mm are within the weld material.

The scatter of the data is clearly more than that of the D9 line (which is completely in parent material).

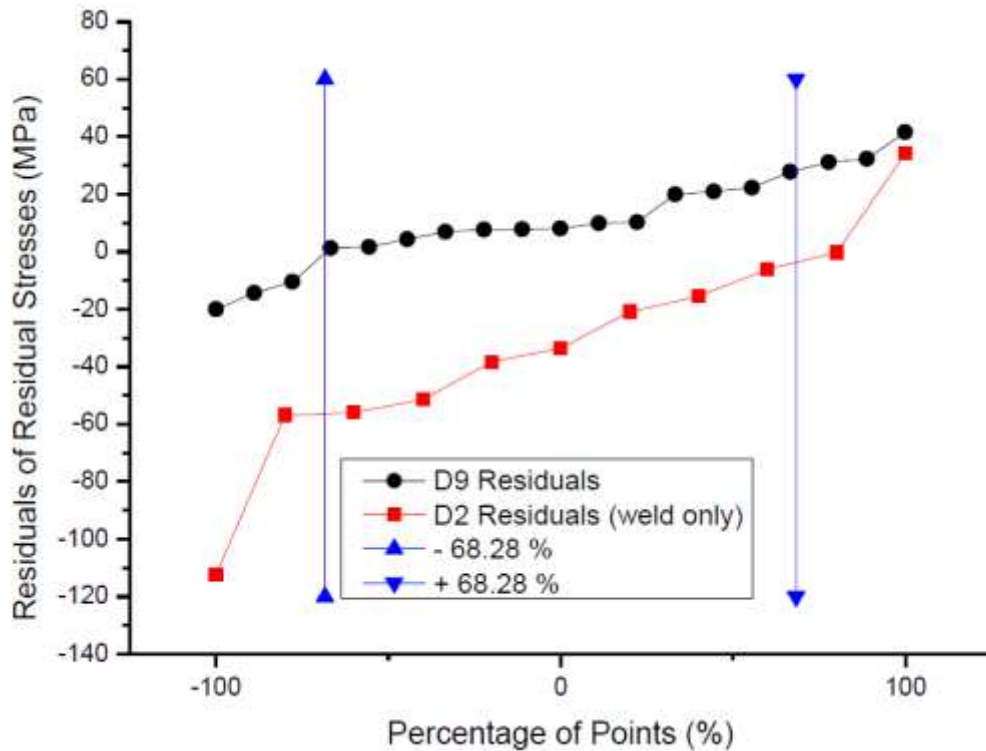
Many measurements were made in NeT-TG4 and this gave a good opportunity to calculate a robust average of all the measurements.

One can take away this robust average from each data set and study the residuals to calculate the actual systematic offsets and random uncertainties (which contains the grain size contribution as well as fitting uncertainties).

It should be noted that the robust average contained measurements on other nominally the same NeT-TG4 specimens, i.e. the 1-1A and 2-1A specimens as well as the 3-1A.

# Analyzing the residuals

Longitudinal direction



$u(\sigma)$

In order to discriminate against outliers in the data, after taking away the appropriate robust average from each data set, the residuals were arranged equidistantly in magnitude order .

Subsequently after scaling the abscissa: -100% to 100%, a linear fit was made between  $\pm 68.28\%$ , corresponding to  $\pm 1$  standard deviation (between the two vertical lines in the figure).

This linear fit gives simultaneously the systematic offset and the total random uncertainty from the gradient.

Of the total 19 points in each direction of the D9 line, 13 points lie within the first standard deviation.

For the D5 and D2 lines, the measurement points between  $z = -40$  and  $40$  mm are within the weld material and only these were considered to estimate the systematic and random uncertainties. This meant that out of a total number of points of 11 in the weld region, 7 points laid within the first standard deviation .

# Comparison of Model with Actual Uncertainties

Parent

Sample	$P$	$u(\sigma_{grain})$ D9 Model [MPa]	$u(\sigma_{model})$ Including fitting uncertainty of 15 [MPa]	$u(\sigma_{actual})$ Random uncertainty from R-fits [MPa]
Black coupon {2 2 2}	0.00377	28.7	32.4	31.0
Black coupon {3 1 1}	0.01131	15.3	21.5	19.7
FRM II (a) D9	0.04691	13.4	20.1	15.8
FRM II (b) D9	0.07065	7.6	16.8	22.8
HZB E3 D9	0.08601	4.3	15.6	14.8

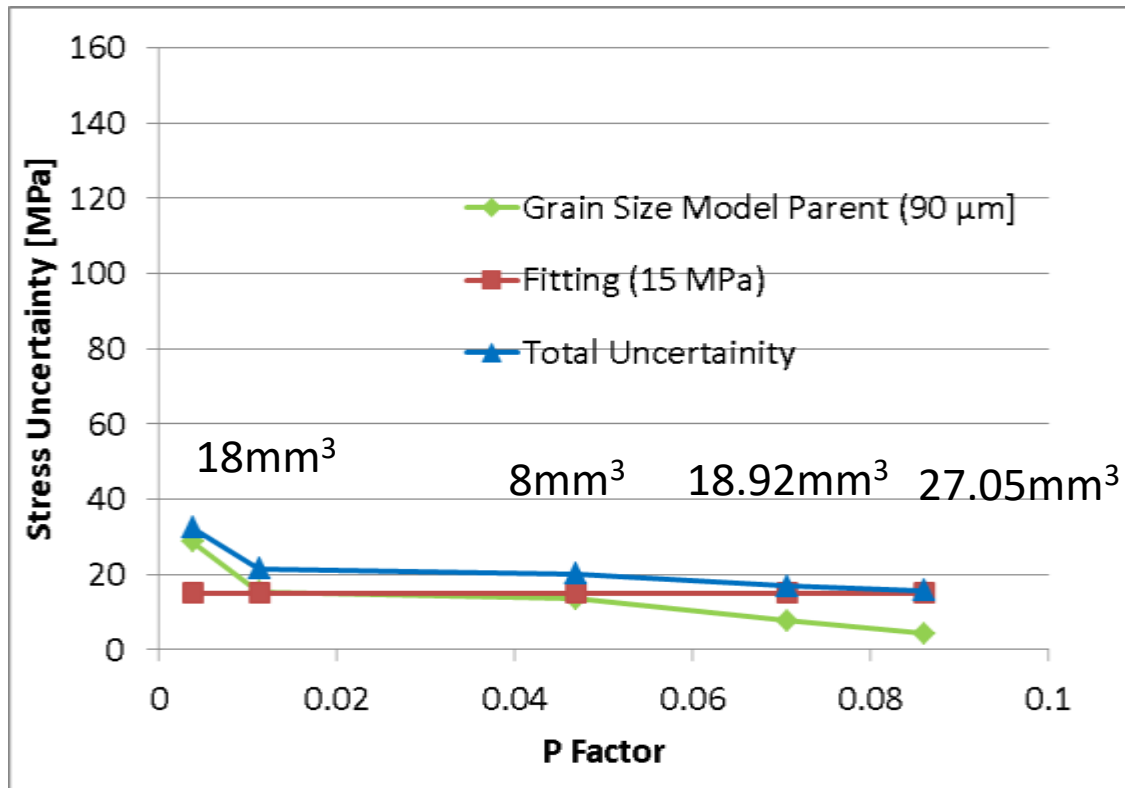
Weld Bottom

Sample	$P$	D5 Model [MPa]	Including fitting uncertainty of 15 [MPa]	Random uncertainty from R-fits [MPa]
Green coupon {2 2 2}	0.00377	84.0	85.4	95.8
Green coupon {3 1 1}	0.01131	43.4	45.9	40.7
FRM II (a) D5	0.04691	37.9	40.8	52.5
FRM II (b) D5	0.07065	21.5	26.2	29.1
HZB E3 D5	0.08601	12.2	19.3	18.2

Weld Top

Sample	$P$	D2 Model [MPa]	Including fitting uncertainty of 15 [MPa]	Random uncertainty from R-fits [MPa]
Red coupon {2 2 2}	0.00377	145.9	146.7	141.6
Red coupon {3 1 1}	0.01131	75.3	76.8	76.9
FRM II (a) D2	0.04691	65.9	67.5	49.7
FRM II (b) D2	0.07065	37.4	40.2	39.2
HZB D2	0.08601	21.1	25.9	33.9

Sample	$P$	$u(\sigma_{grain})$ D9 Model [MPa]	$u(\sigma_{model})$ Including fitting uncertainty of 15 [MPa]	$u(\sigma_{actual})$ Random uncertainty from R-fits [MPa]
Black coupon {2 2 2}	0.00377	28.7	32.4	31.0
Black coupon {3 1 1}	0.01131	15.3	21.5	19.7
FRM II (a) D9	0.04691	13.4	20.1	15.8
FRM II (b) D9	0.07065	7.6	16.8	22.8
HZB E3 D9	0.08601	4.3	15.6	14.8



$$u(\sigma) \approx (u(\sigma_{fitting})^2 + u(\sigma_{grain})^2)^{1/2}$$

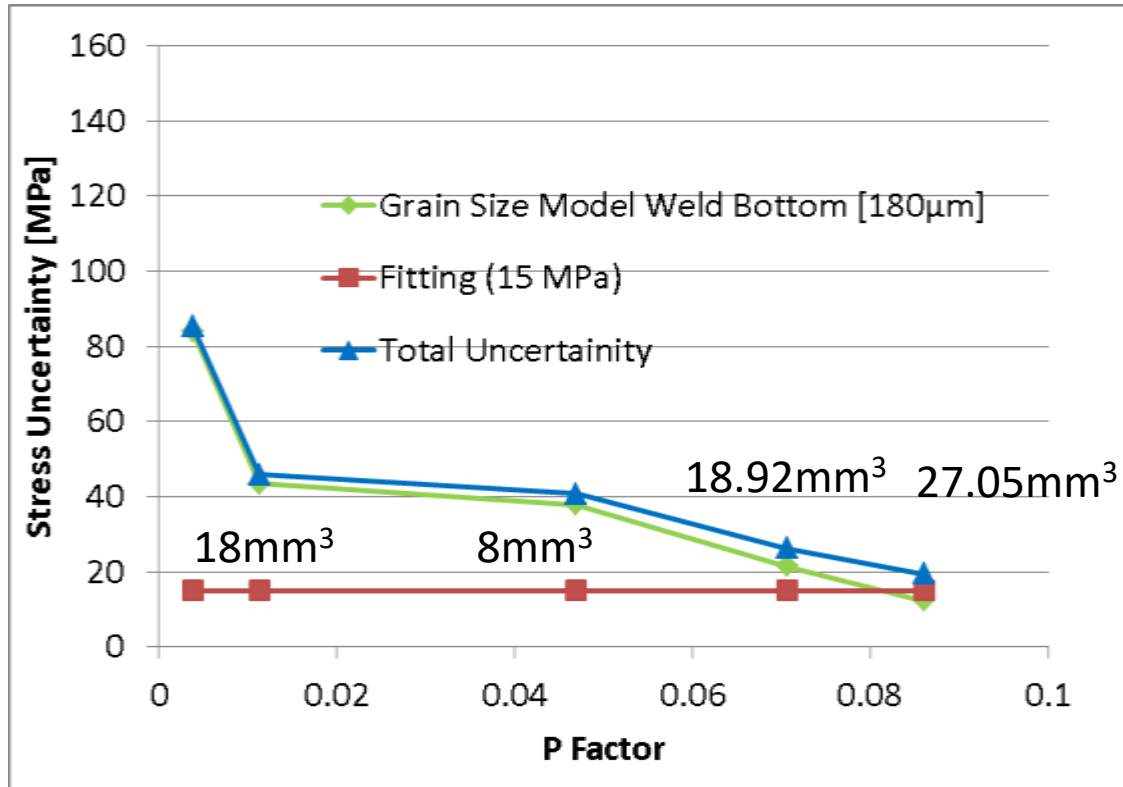
↑  
=15 MPa

$$u(2\theta_{grain}) \approx \frac{0.5 * SD_{Gauss}}{(N_{DG})^{1/2}}$$


$$N_{DG} \approx \left( P * \frac{gv}{(S_G)^3} \right) \quad S_G = 90\mu\text{m}$$

Weld Bottom

Sample	$P$	$u(\sigma_{grain})$ D5 Model [MPa]	$u(\sigma_{model})$ Including fitting uncertainty of 15 [MPa]	$u(\sigma_{actual})$ Random uncertainty from R-fits [MPa]
Green coupon {2 2 2}	0.00377	84.0	85.4	95.8
Green coupon {3 1 1}	0.01131	43.4	45.9	40.7
FRM II (a) D5	0.04691	37.9	40.8	52.5
FRM II (b) D5	0.07065	21.5	26.2	29.1
HZB E3 D5	0.08601	12.2	19.3	18.2



$$u(\sigma) \approx (u(\approx_{fitting})^2 + u(\approx_{grain})^2)^{1/2}$$

  
 =15 MPa

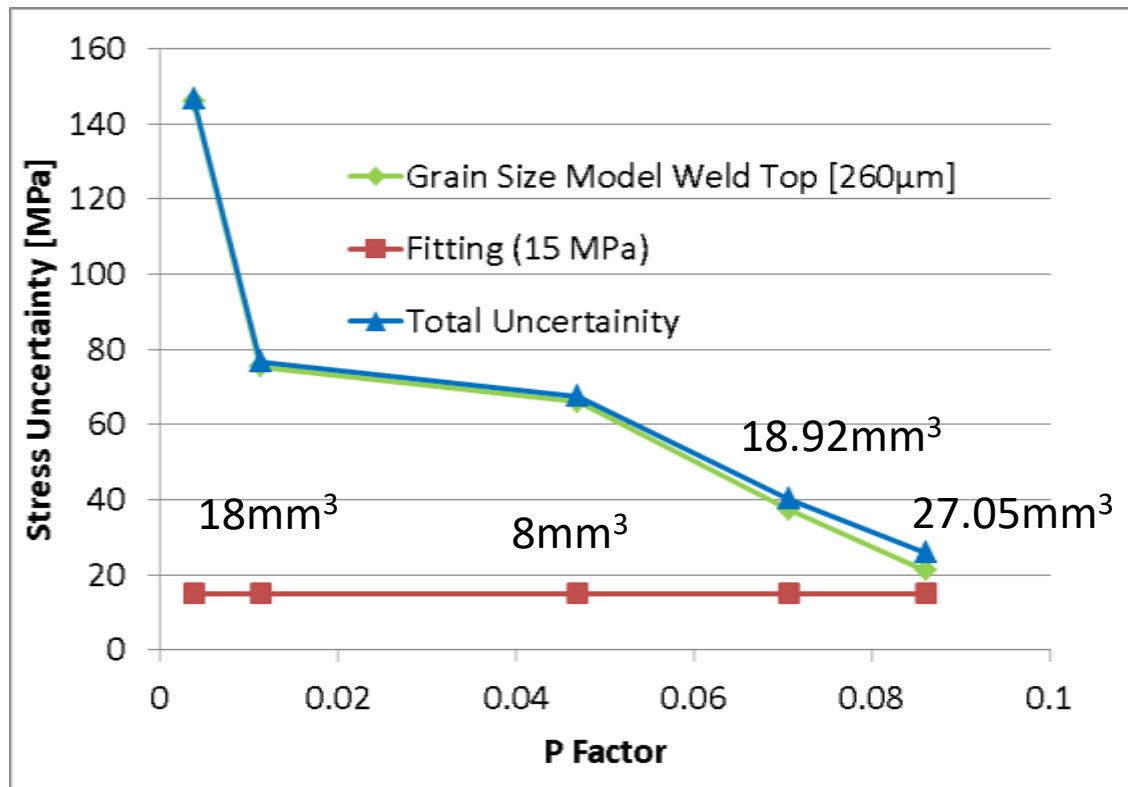
$$u(2\theta_{grain}) \approx \frac{0.5 * SD_{Gauss}}{(N_{DG})^{1/2}}$$

$$N_{DG} \approx \left( P * \frac{gv}{(S_G)^3} \right) \quad S_G = 180\mu m$$

$u(\sigma_{grain})$  $u(\sigma_{model})$  $u(\sigma_{actual})$ 

Weld Top

Sample	$P$	D2 Model [MPa]	Including fitting uncertainty of 15 [MPa]	Random uncertainty from R-fits [MPa]
Red coupon {2 2 2}	0.00377	145.9	146.7	141.6
Red coupon {3 1 1}	0.01131	75.3	76.8	76.9
FRM II (a) D2	0.04691	65.9	67.5	49.7
FRM II (b) D2	0.07065	37.4	40.2	39.2
HZB D2	0.08601	21.1	25.9	33.9

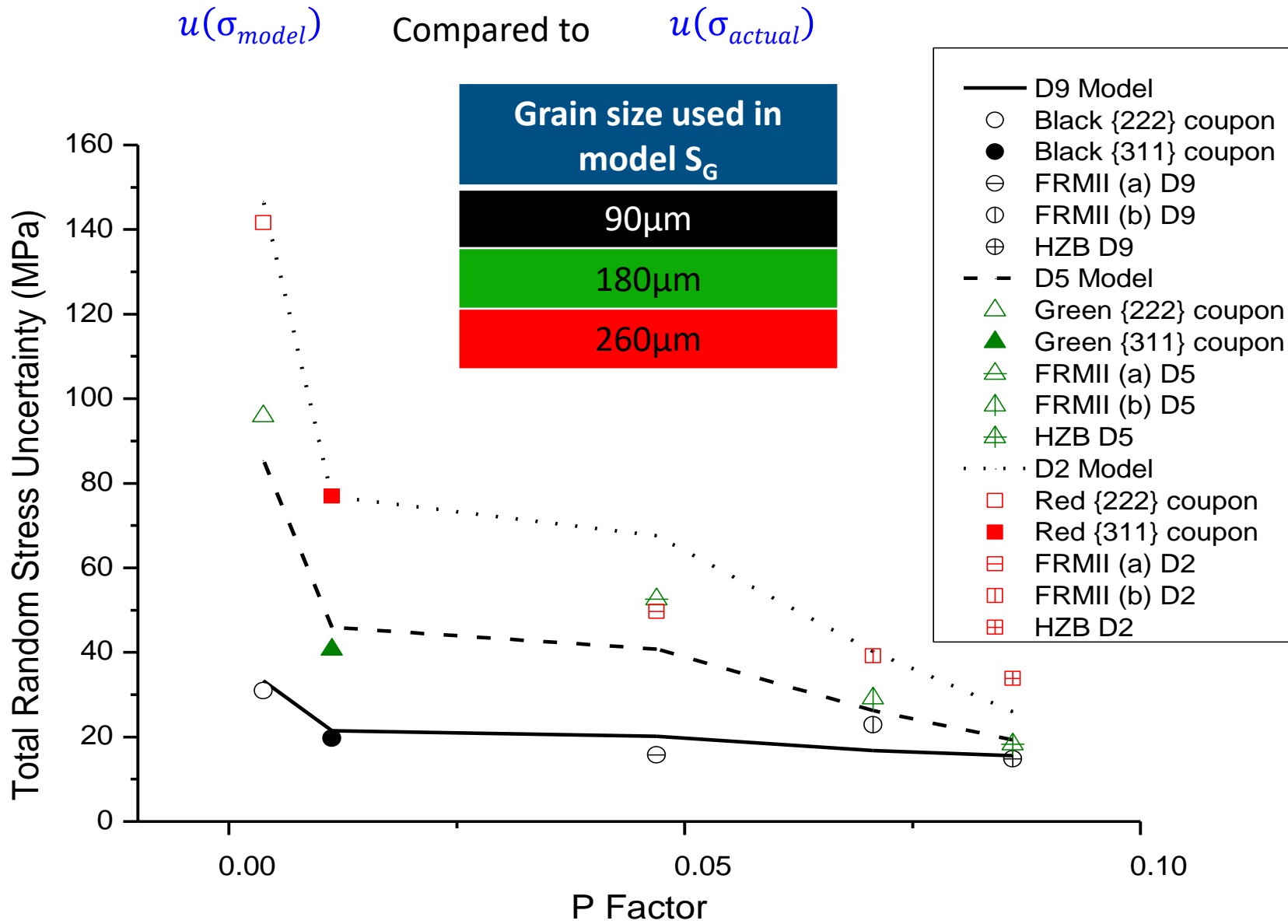


$$u(\sigma) \approx (u(\sigma_{fitting})^2 + u(\sigma_{grain})^2)^{1/2}$$

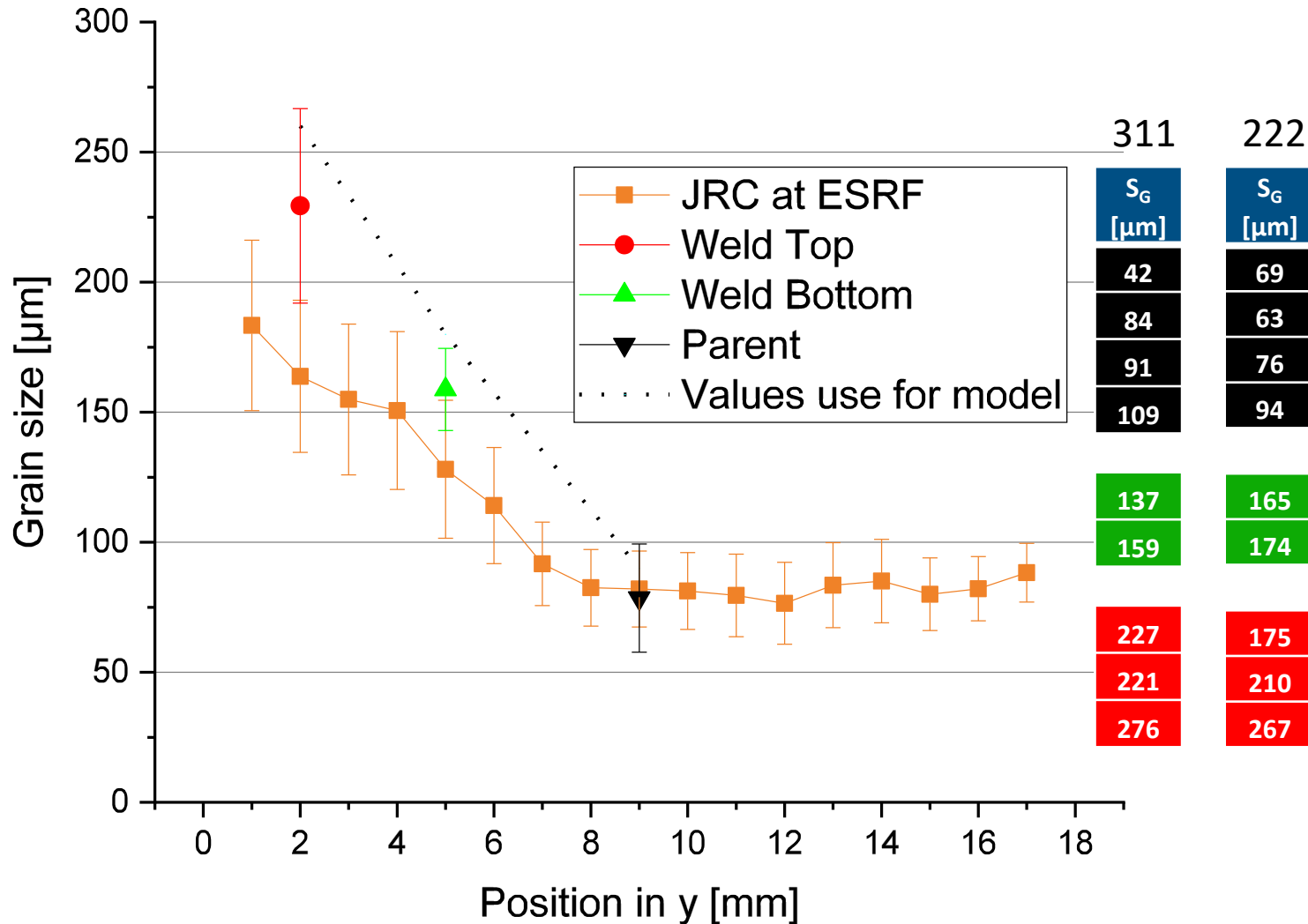
↑  
=15 MPa

$$u(2\theta_{grain}) \approx \frac{0.5 * SD_{Gauss}}{(N_{DG})^{1/2}}$$

$$N_{DG} \approx \left( P * \frac{gv}{(S_G)^3} \right) \quad S_G = 260\mu m$$



# Grain size



JRC at ESRF (ID15a, spiral slit set up). High energy synchrotron X-rays (using 5 peaks)



# Uncertainty in the uncertainties

The peak fitting uncertainty is often **not enough** to describe completely the true random uncertainty of a neutron strain measurement and resultant stress determinations.

$$\varepsilon = \frac{\sin\theta_0}{\sin\theta} - 1 \longrightarrow u(\varepsilon) = \frac{1}{\tan\theta_0} \left[ u(\theta_{fitting})^2 + u(\theta_{0-fitting})^2 \right]^{\frac{1}{2}} \quad \text{The traditional way}$$

Detecting not enough diffracting grains also contributes to the random uncertainty.

A simple model is needed to estimate the **extra random uncertainty contribution due to the so-called grain size statistics.**

$$u(\varepsilon) = \frac{1}{\tan\theta_0} \left[ u(\theta_{fitting})^2 + u(\theta_{grain})^2 + u(\theta_{0-fitting})^2 + u(\theta_{0-grain})^2 \right]^{\frac{1}{2}} \quad \text{The way we should do it}$$

Either by multiple measurement or 'modeling'

# Remarks and Conclusions

Single shot measurement of 'main sample': Only gives fitting uncertainty, however is the normal measurement practice

Fitting uncertainty is time dependent whereas the uncertainty due to grain size is dependent on the number of detected diffracting grains

Propagation of only the fitting uncertainty of Bragg peaks is not enough with 'large grains'

Multiple Measurement of 'main sample': Time constraints, Expensive Beam Time, not the normal practice

Multiple Measurement of 'Representative References': Can be quick and give information about grain-size uncertainty in the 'main sample' and used to estimate the extra uncertainty due to grain size.

Also a priori knowledge of grain size can also be directly used in the model to estimate the extra uncertainty needed to add to the fitting uncertainty.

Thank you