Neutron Grating Interferometry
Part II

Quantitative DFI

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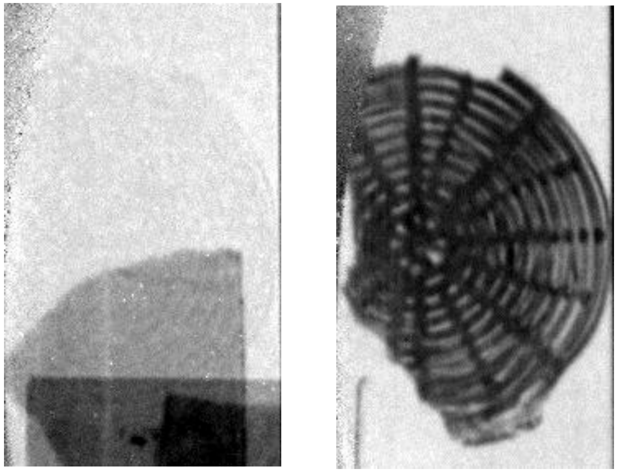
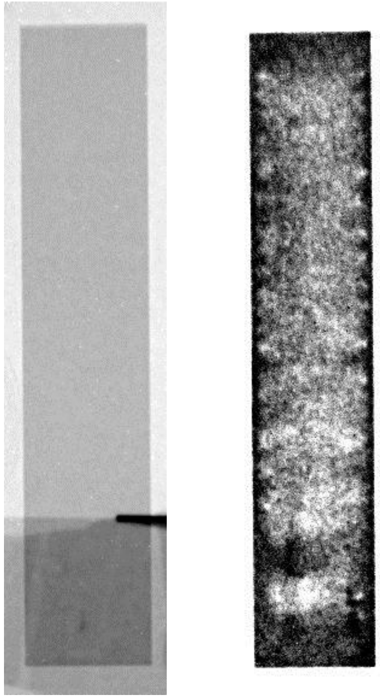
Dark Field Images

nGI reveals scattering

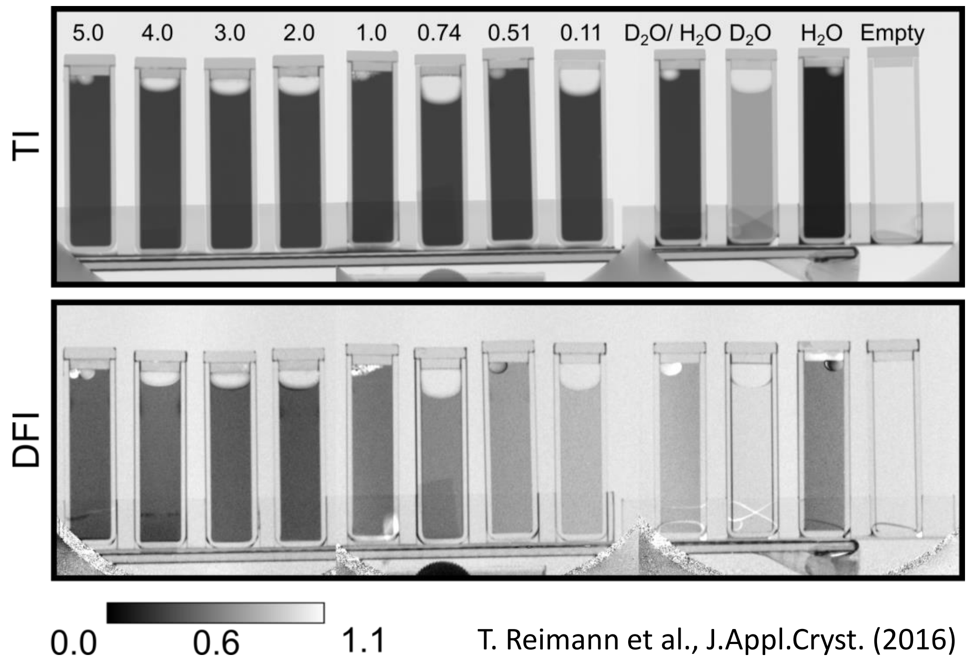
- from small structures (~ μm)
- under very small angles





What does that mean exactly ?

Quantitative DFI: Size Matters

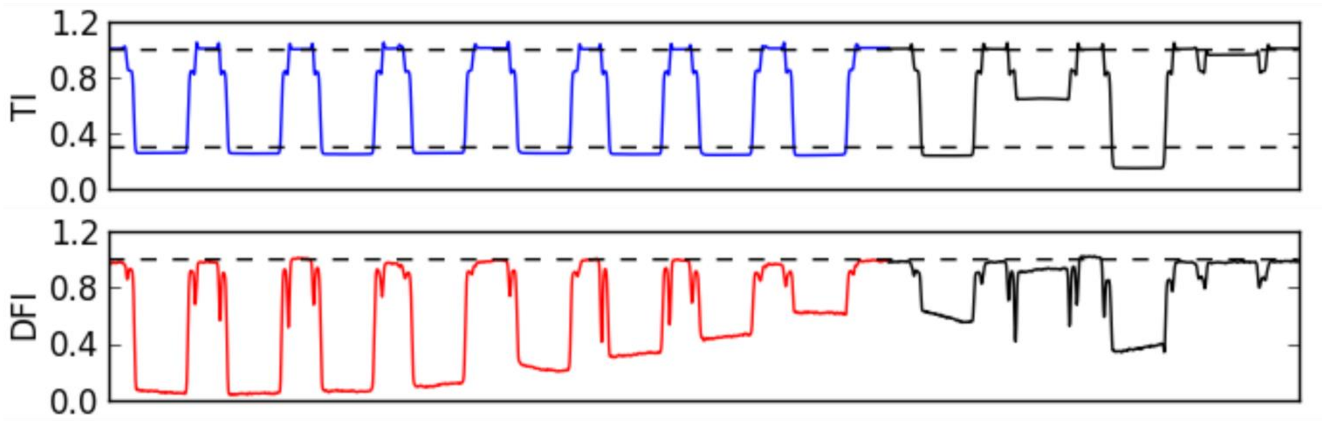


T. Reimann et al., J.Appl.Cryst. (2016)

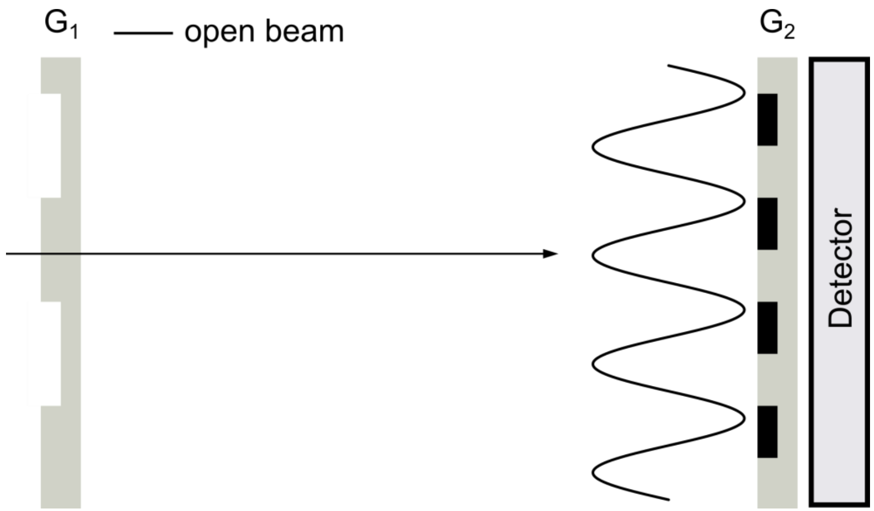
Polysterene Colloids in Solution

- varying particle diameter
- constant volume fraction

- constant absorbtion
- varying DFI value



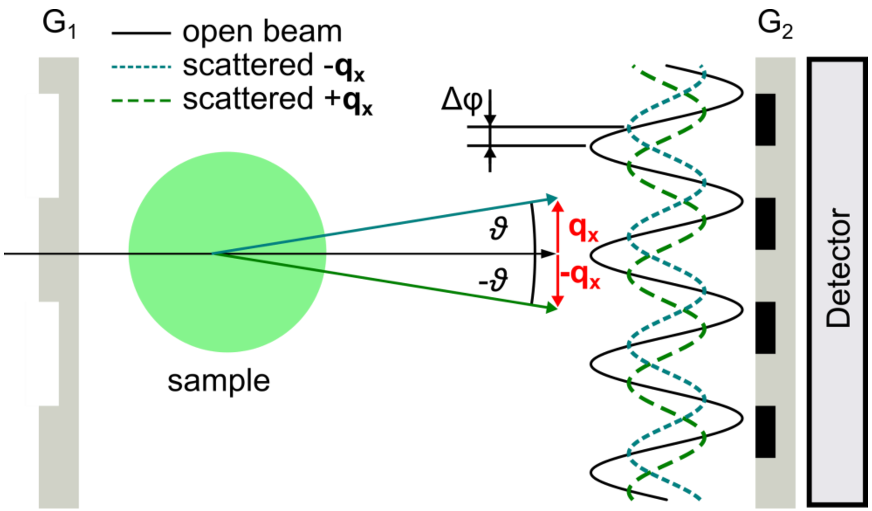
Model Case: One Single Scattering Angle



without sample:

$$V_{ob} = \cos(x)$$

Model Case: One Single Scattering Angle



without sample:

$$V_{ob} = \cos(x)$$

with sample:

$$V_s = \frac{1}{2}\cos(x+\vartheta) + \frac{1}{2}\cos(x-\vartheta)$$

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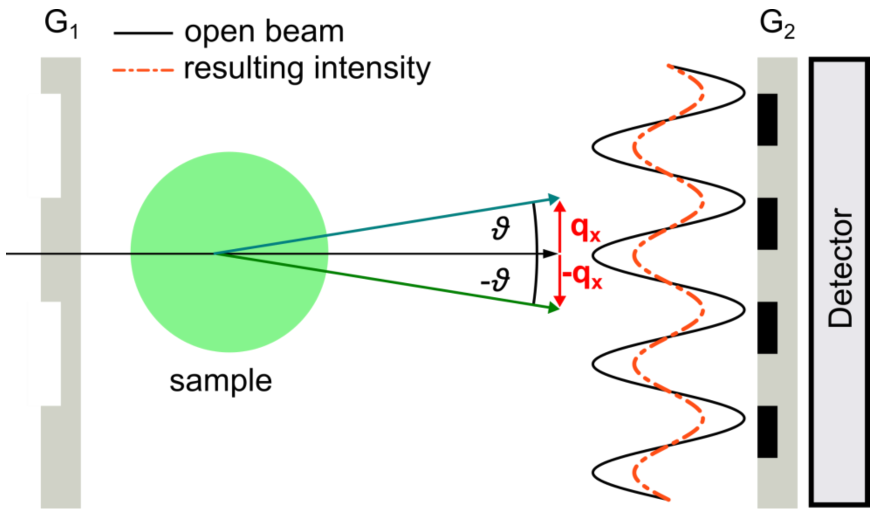


without sample:

$$V_{ob} = \cos(x)$$

with sample:

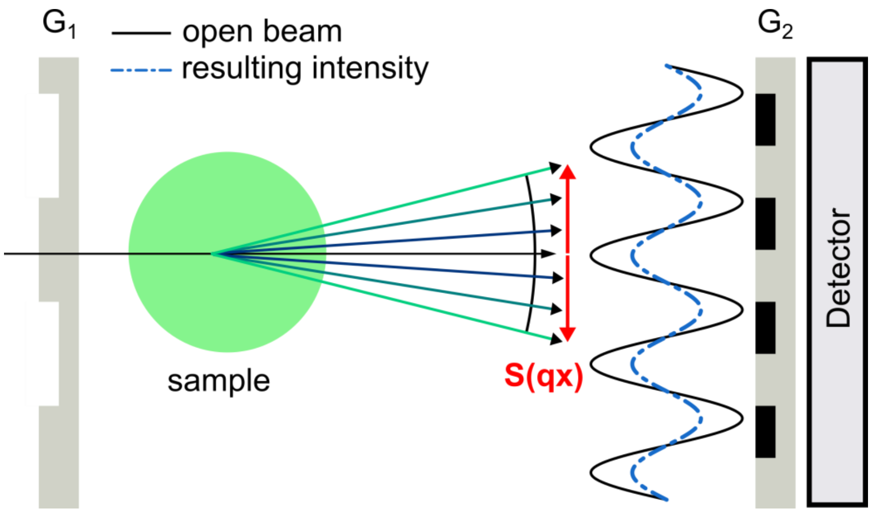
$$V_s = \frac{1}{2}\cos(x+\vartheta) + \frac{1}{2}\cos(x-\vartheta)$$

$$V_s = \cos(x)\cos(\xi_{GI} q_x)$$

$$\vartheta = \xi_{GI} q_x \qquad \xi_{GI} = \lambda \frac{L_{s,eff}}{p_2}$$

$$DFI = \frac{V_s}{V_{ob}} = \cos(\xi_{GI} q_x)$$

Real Case: Distribution of Scattering Angles



$$DFI = \cos(\xi_{GI} q_x)$$

$$DFI = \int S(q_x) \cos(\xi_{GI} q_x)\, dq_x$$

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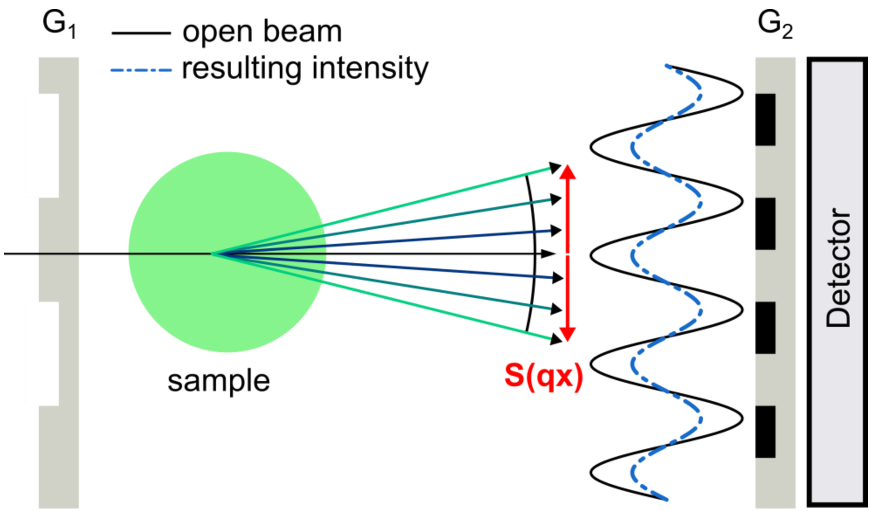

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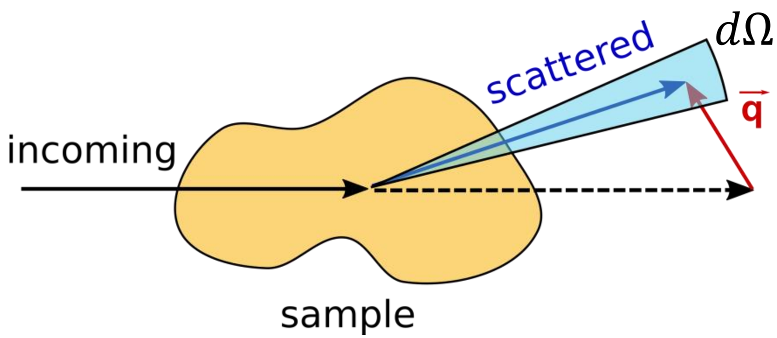
$S(q_x)$
"probability of scattering a neutron with a certain momentum transfer q_x "

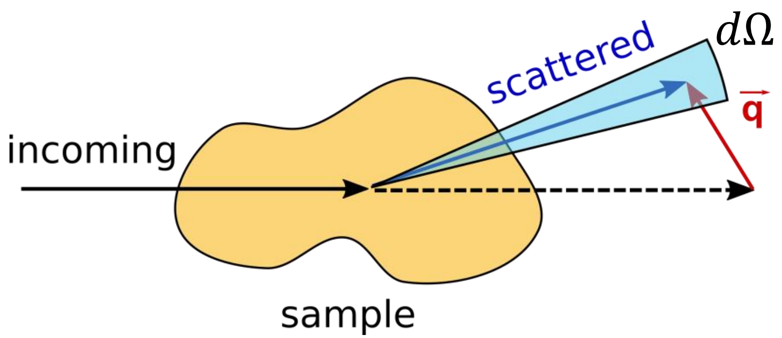
What defines $\mathbf{S(q_x)}$?

Reciprocal Space: the differential cattering cross section (looking at momenum)



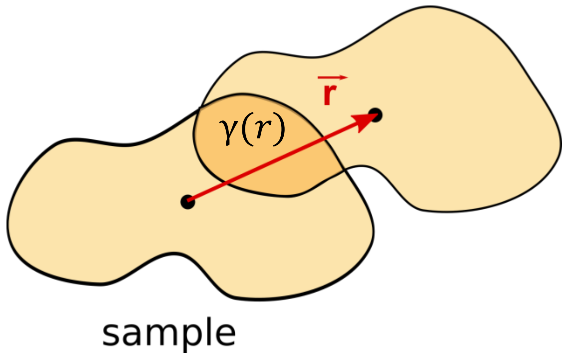
$$\frac{d\sigma(q)}{d\Omega} = \frac{\text{neutrons scattered into } d\Omega}{\text{incoming neutrons per unit area}}$$

Reciprocal Space: the differential cattering cross section (looking at momenum)



$$\frac{d\sigma(q)}{d\Omega} = \frac{\text{neutrons scattered into } d\Omega}{\text{incoming neutrons per unit area}}$$

Real space: the pair correlation function (looking at space coordinates)

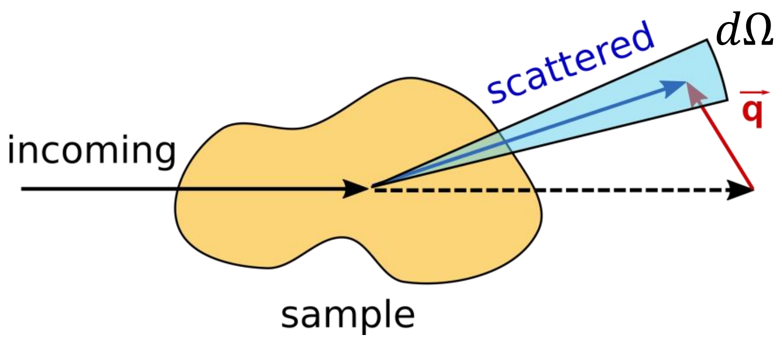


$$\gamma(r) = \int_V \Delta\rho(R)\Delta\rho(R + r)dR$$

Distribution of scsattering strength

$S(q_x)$: A Bit of Scattering Theory

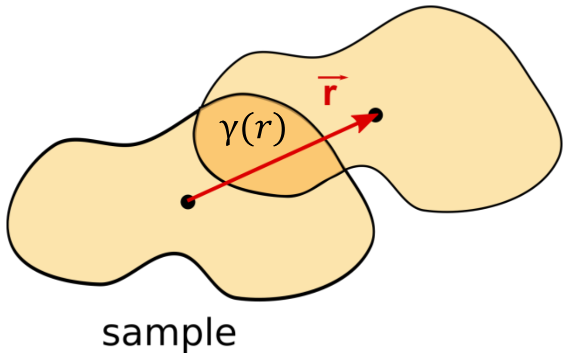
Reciprocal Space: the differential cattering cross section (looking at momenum)



$$\frac{d\sigma(q)}{d\Omega} = \frac{\text{neutrons scattered into } d\Omega}{\text{incoming neutrons per unit area}}$$

fourier transformation

Real space: the pair correlation function (looking at space coordinates)

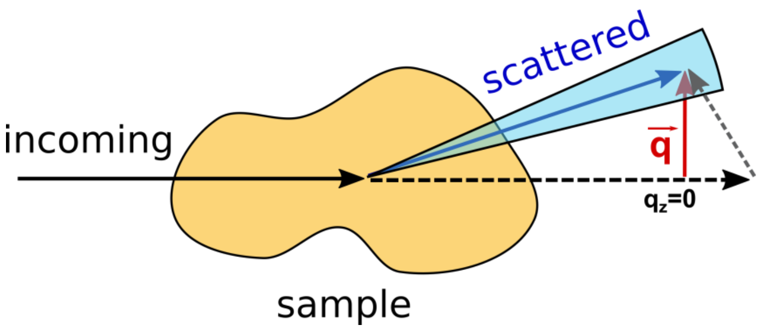


$$\gamma(r) = \int_V \Delta\rho(R)\Delta\rho(R+r)dR$$

Distribution of scsattering strength

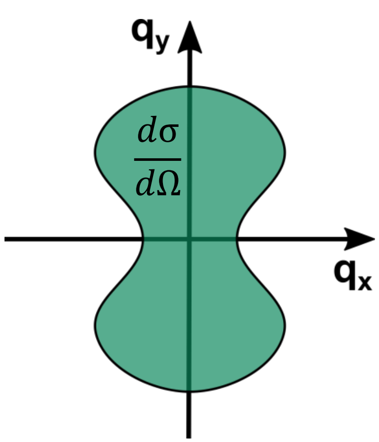
$S(q_x)$: Reducing the Dimensions

Step 1: no scattering along the beam direction $q_z = 0$



$$\frac{d\sigma(q)}{d\Omega} \rightarrow \frac{d\sigma(q_x, q_y, 0)}{d\Omega}$$

$$\Rightarrow \quad G(x, y) = \int \gamma(x, y, z)\, dz$$



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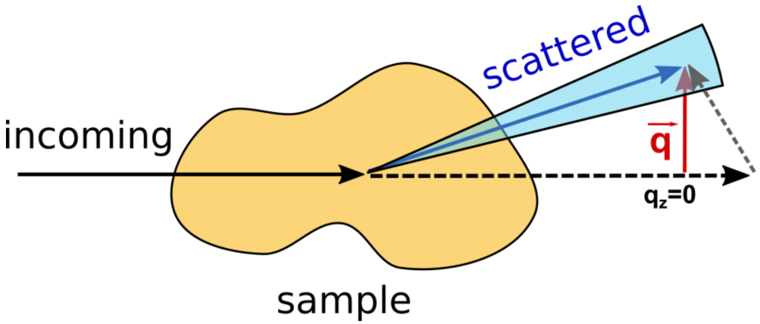
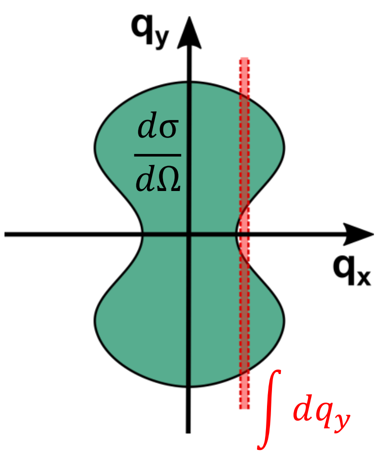


$$\frac{d\sigma(q)}{d\Omega} \rightarrow \frac{d\sigma(q_x, q_y, 0)}{d\Omega}$$

$$\Rightarrow \quad G(x,y) = \int \gamma(x,y,z)\, dz$$

Step 2: projection along the grating lines $q_y = -\infty \rightarrow +\infty$



$$\left(\frac{d\sigma(q_x)}{d\Omega}\right)_{slit} = \int \frac{d\sigma(q_x, q_y, 0)}{d\Omega} dq_y$$

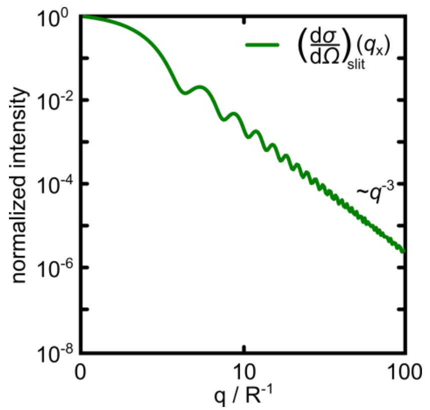
$$\Rightarrow \quad G(x,y) \rightarrow G(x,0)$$

Reciprocal Space:

$$S(q_x) \approx \left(\frac{d\sigma(q_x)}{d\Omega}\right)_{slit}$$

$$q_z = 0$$
$$q_y = -\infty \to +\infty$$

$$DFI = \int S(q_x) \cos(\xi_{GI} q_x)\, dq_x$$



T. Reimann, Ph.D. Thesis (2016)

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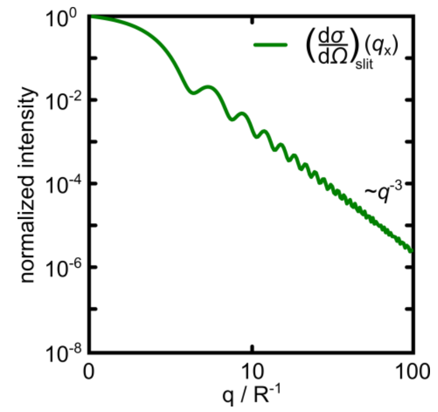
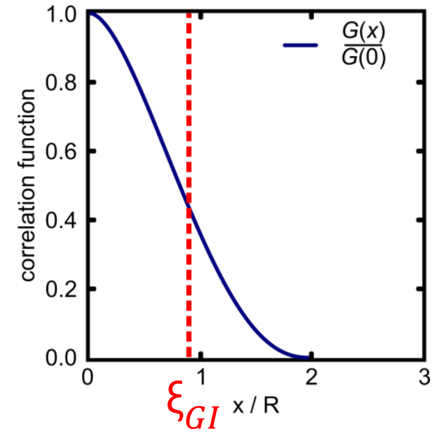


Real Space:

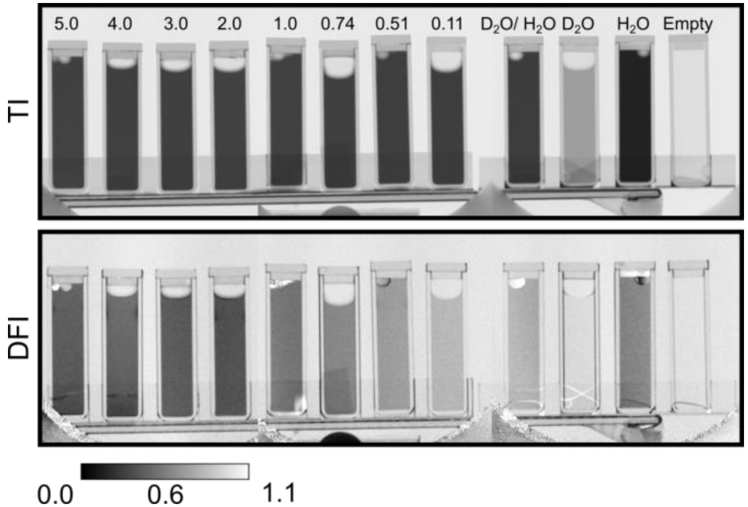
$$DFI = exp\left[\Sigma t \left(\frac{G(\xi_{GI})}{G(0)} - 1\right)\right]$$



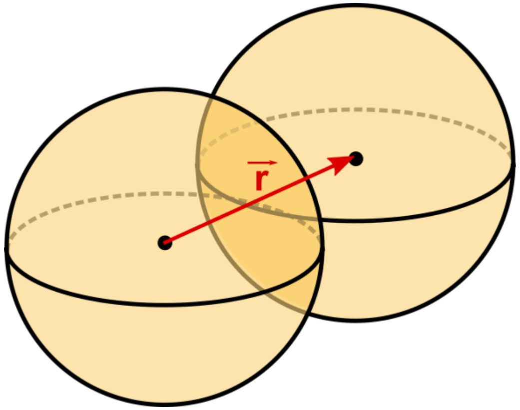
Total scattering crossection	$\Sigma = (\Delta\rho)^2 \lambda^2 \Phi_v \frac{G(0)}{\gamma(0)}$
sample thickness	t
nGI correlation length	ξ_{GI}

T. Reimann, Ph.D. Thesis (2016)

Polysterene Colloids in Solution



Pair Correlation Function



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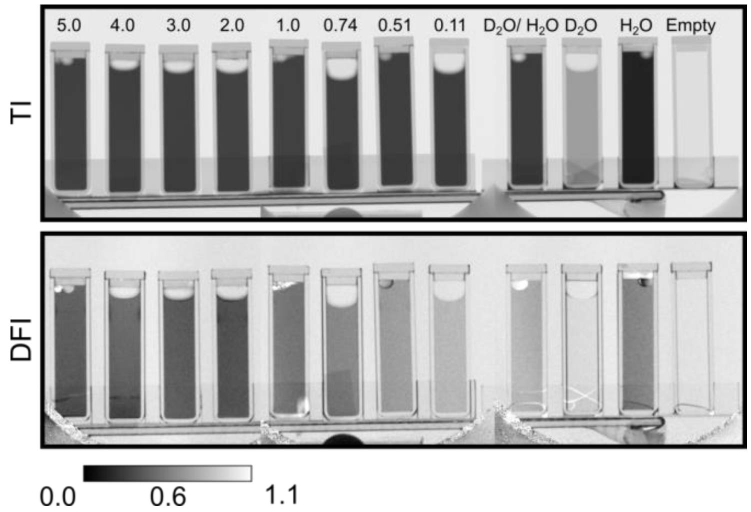
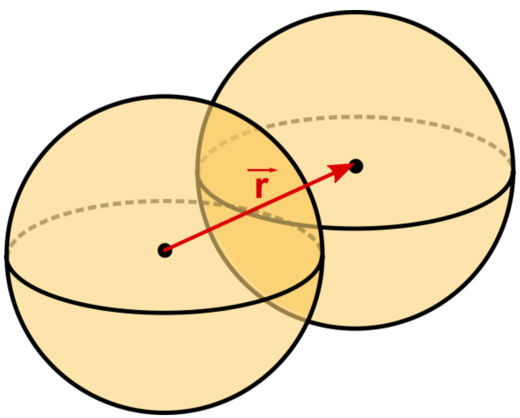


Pair Correlation Function



calculate:		**fourier transform:**
$\gamma(x,y,z)$	→	$\frac{d\sigma(q_x,q_y,q_z)}{d\Omega}$
$G(x,y)$	→	$\frac{d\sigma(q_x,q_y,0)}{d\Omega}$
$G(x,0)$	→	$\left(\frac{d\sigma(q_x)}{d\Omega}\right)_{slit}$
Real Space		Reciprocal Space

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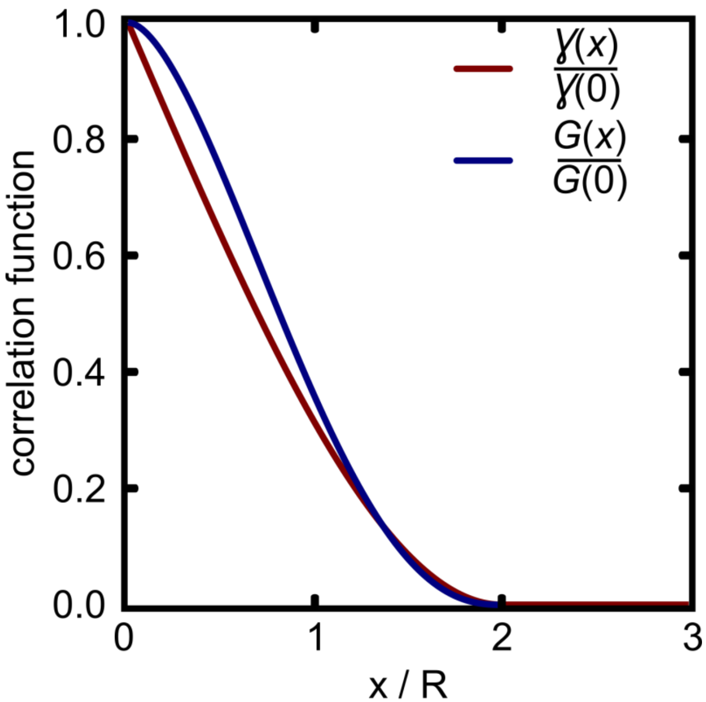
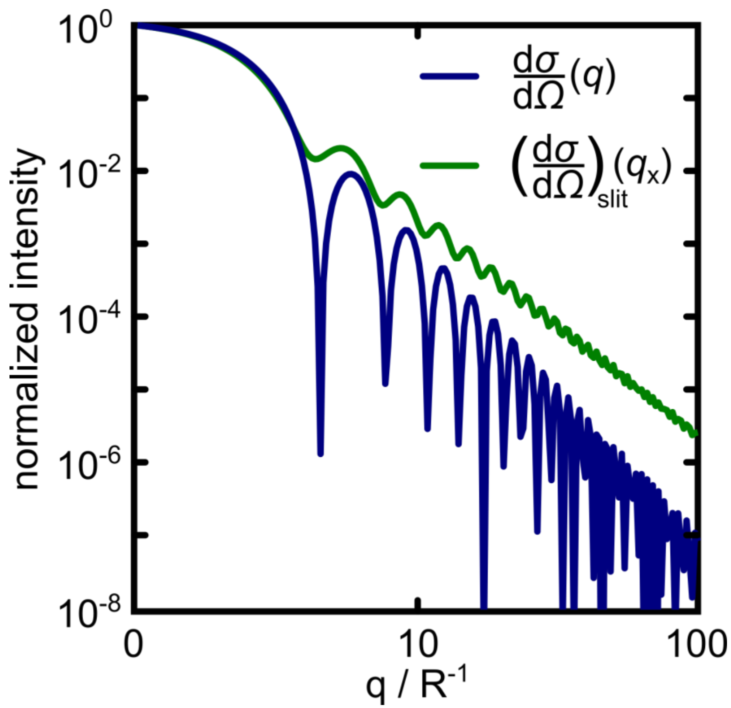
Polysterene Colloids in Water

Real Space:
Pair Correlation function



Reciprocal Space:
Scattering cross section

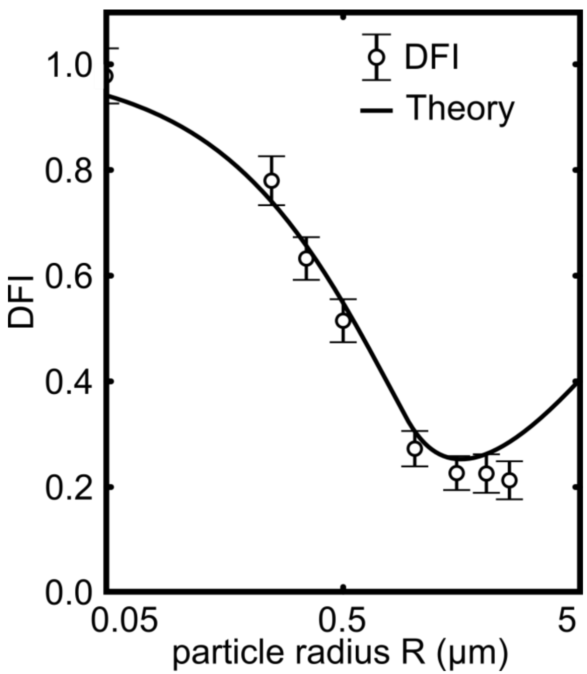


T. Reimann et al., J.Appl.Cryst. (2016)

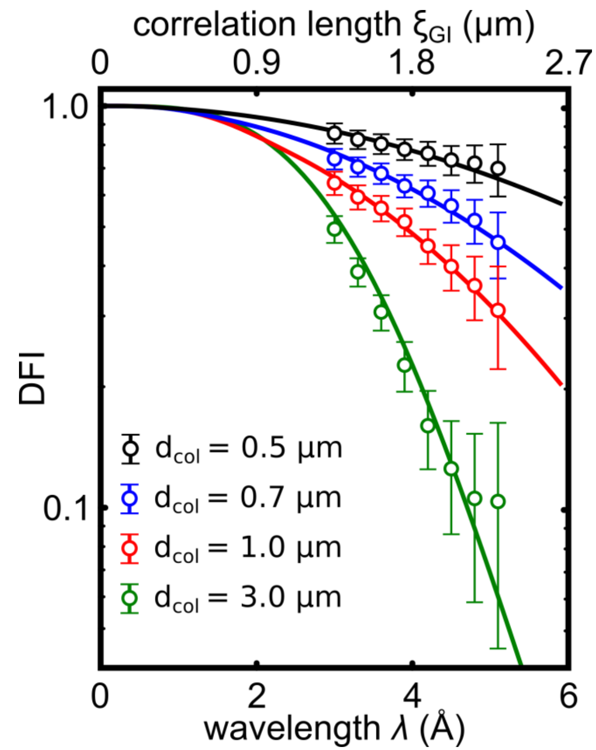
Polysterene Colloids in Water

$$DFI = exp\left[\Sigma t\left(\frac{G(\xi_{GI})}{G(0)} - 1\right)\right]$$

R **dependence:**
- $G = G(R)$
- $\Sigma = \Sigma(R)$



λ dependence:
- $\xi_{GI} = \xi_{GI}(\lambda)$
- $\Sigma = \Sigma(\lambda)$



T. Reimann et al., J.Appl.Cryst. (2016)

DFI depends only on one value of the pair correlation function $G(\xi_{GI})$

More information can be gained by:

Changing G

↓

Different particle sizes

↓

Only possible in rare cases

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Changing ξ_{GI}

Change the setup
or
Sample position

Various geometric constraints
Tedious callibrations

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More information can be gained by:

Changing G → Different particle sizes → Only possible in rare cases

Changing ξ_{GI}

- Change the setup or Sample position → Various geometric constraints; Tedious callibrations
- Vary the wavelength → Only a small range is accessible

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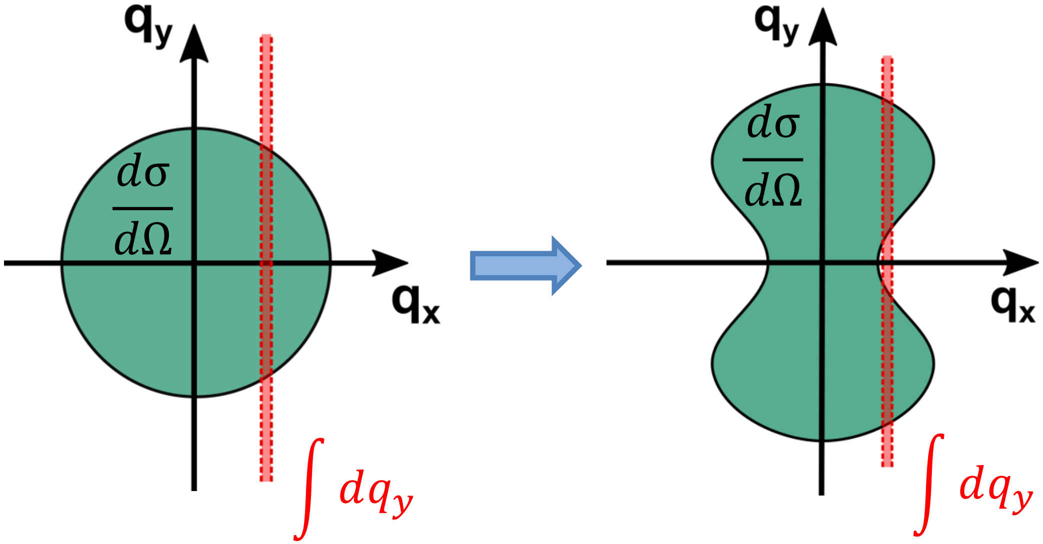
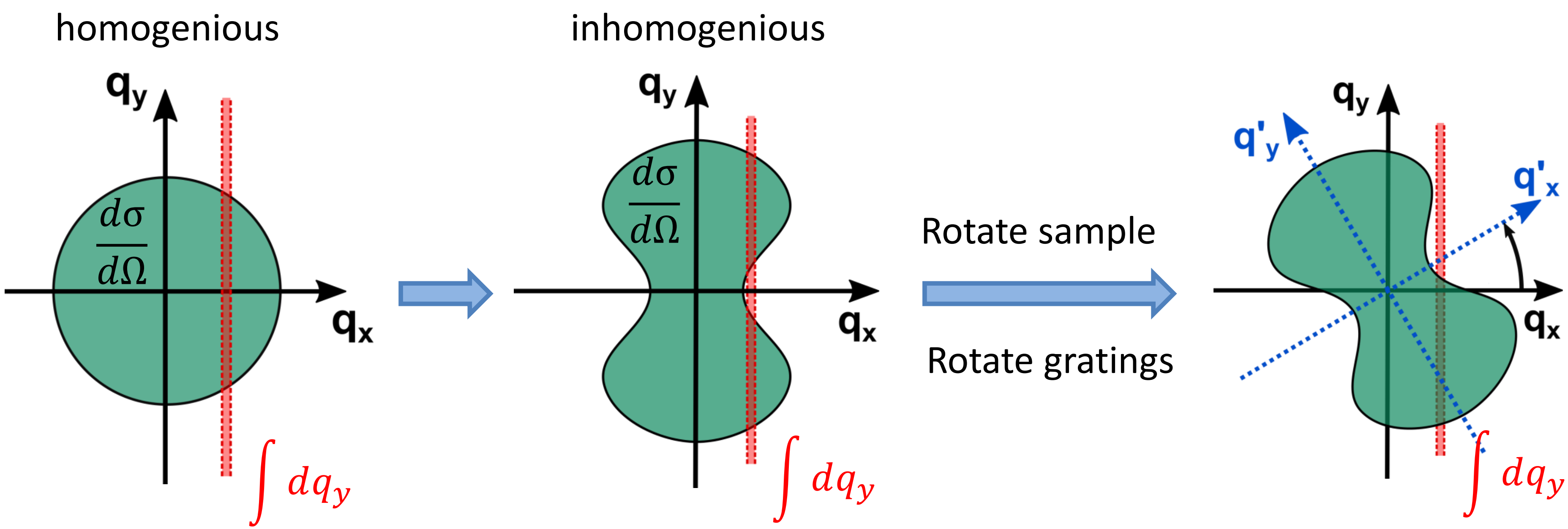
Inhomogeneous Scattering

homogenious

inhomogenious

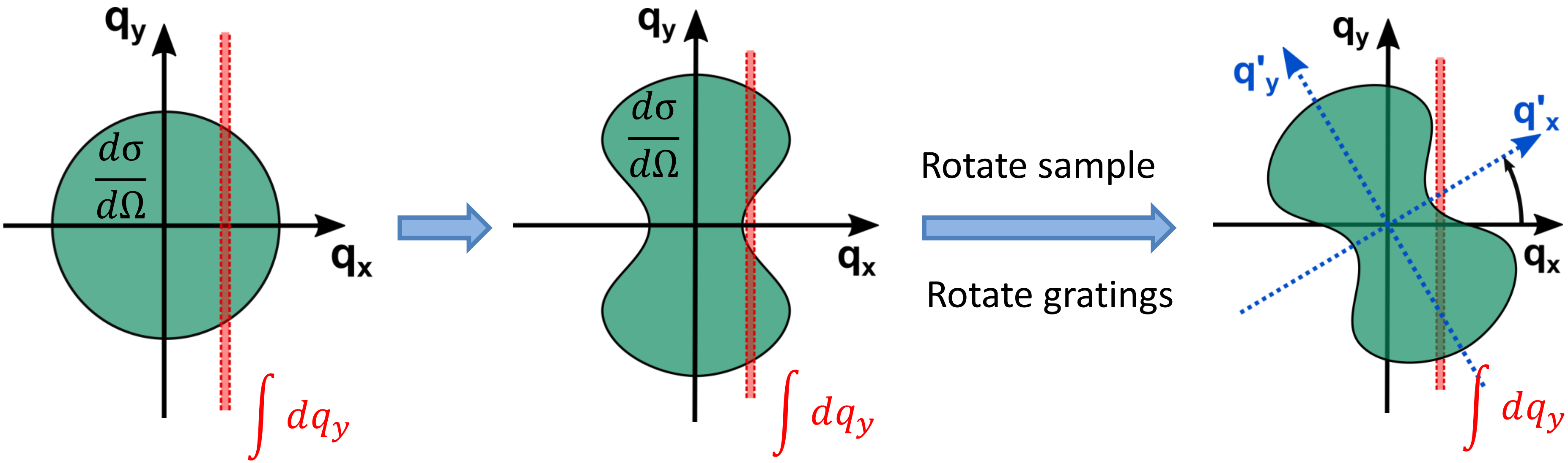




$\left(\frac{d\sigma(q_x)}{d\Omega}\right)_{slit}$ depends on the relative orientation of sample and gratings

Inhomogeneous Scattering

homogenious inhomogenious

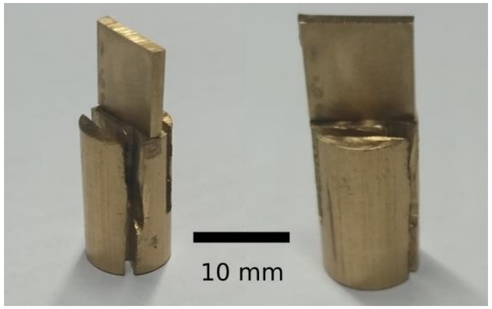


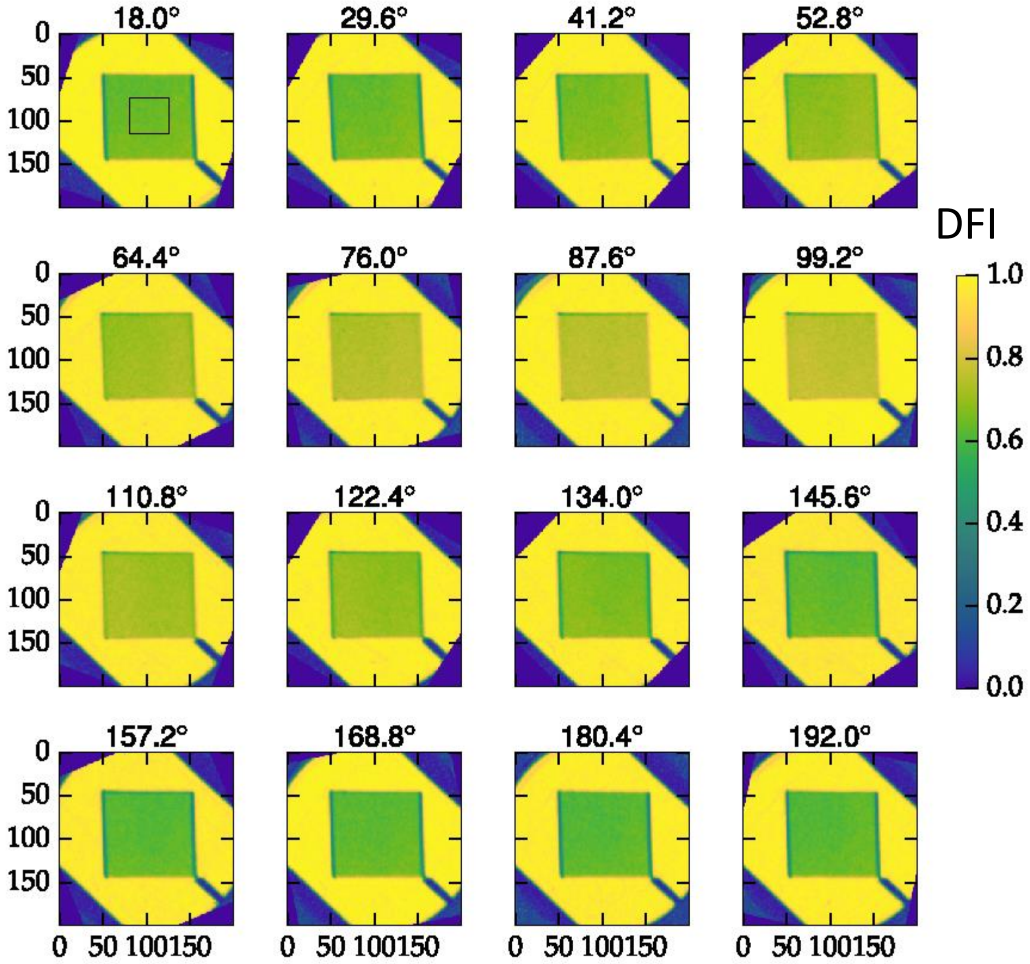
$\left(\frac{d\sigma(q_x)}{d\Omega}\right)_{slit}$ depends on the relative orientation of sample and gratings

$$DFI(\omega) = exp\left[\Sigma t\left(\frac{G(\xi_{GI}\cos(\omega), -\xi_{GI}\sin(\omega))}{G(0)} - 1\right)\right]$$

Anisotropic scattering in Brass

Extrusion Moulded Brass:
anisotropic
- production
- crystallites
- scattering





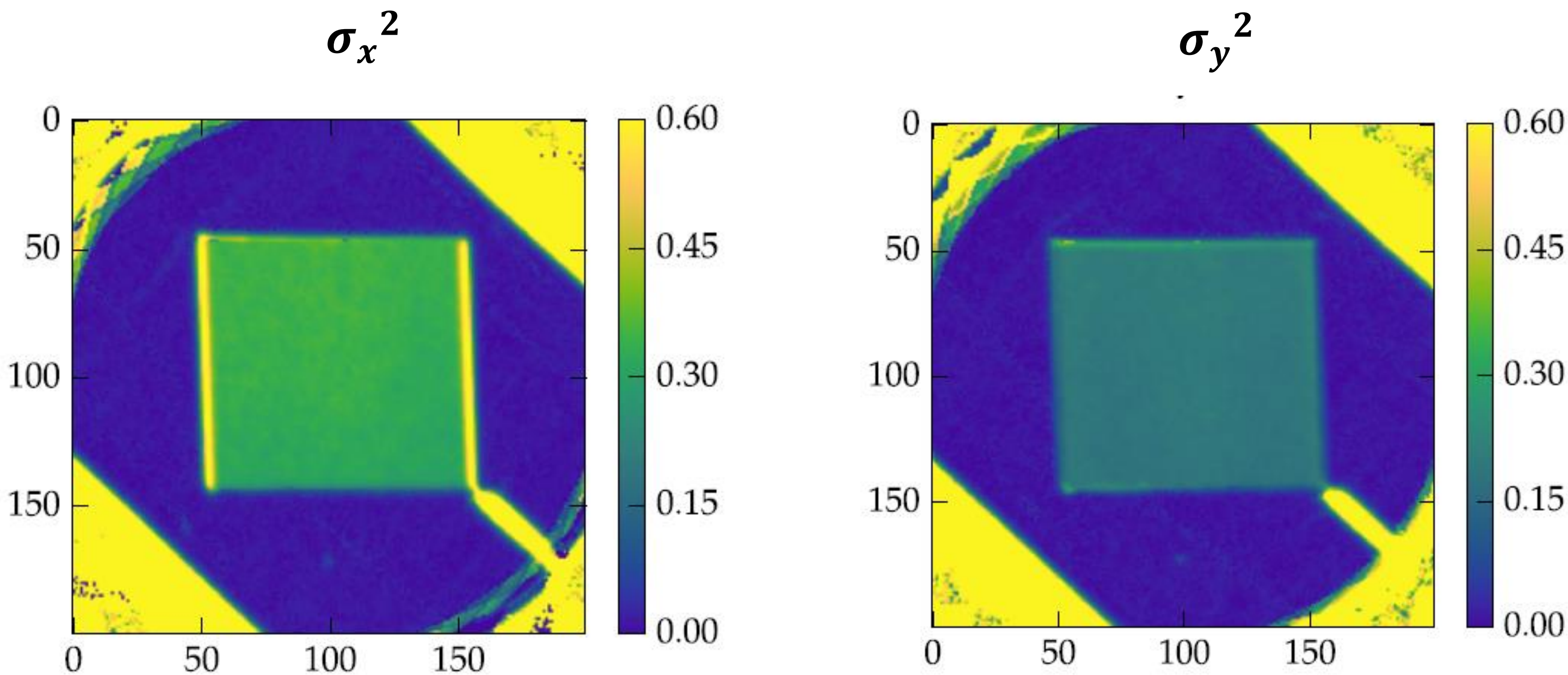
T. Neuwirth, Master Thesis (2017)

Anisotropic scattering in Brass

Different distribution of crystallite size in x and y direction

➡ Bi gaussian scattering cross section

$$DFI(\omega) = exp\left[-\frac{\xi_{GI}}{2}\left(\sigma_x{}^2 + \left(\sigma_y{}^2 - \sigma_x{}^2\right) sin^2(\omega - \varphi)\right)\right] \qquad \sigma_{an} = \sigma_x{}^2 - \sigma_y{}^2$$



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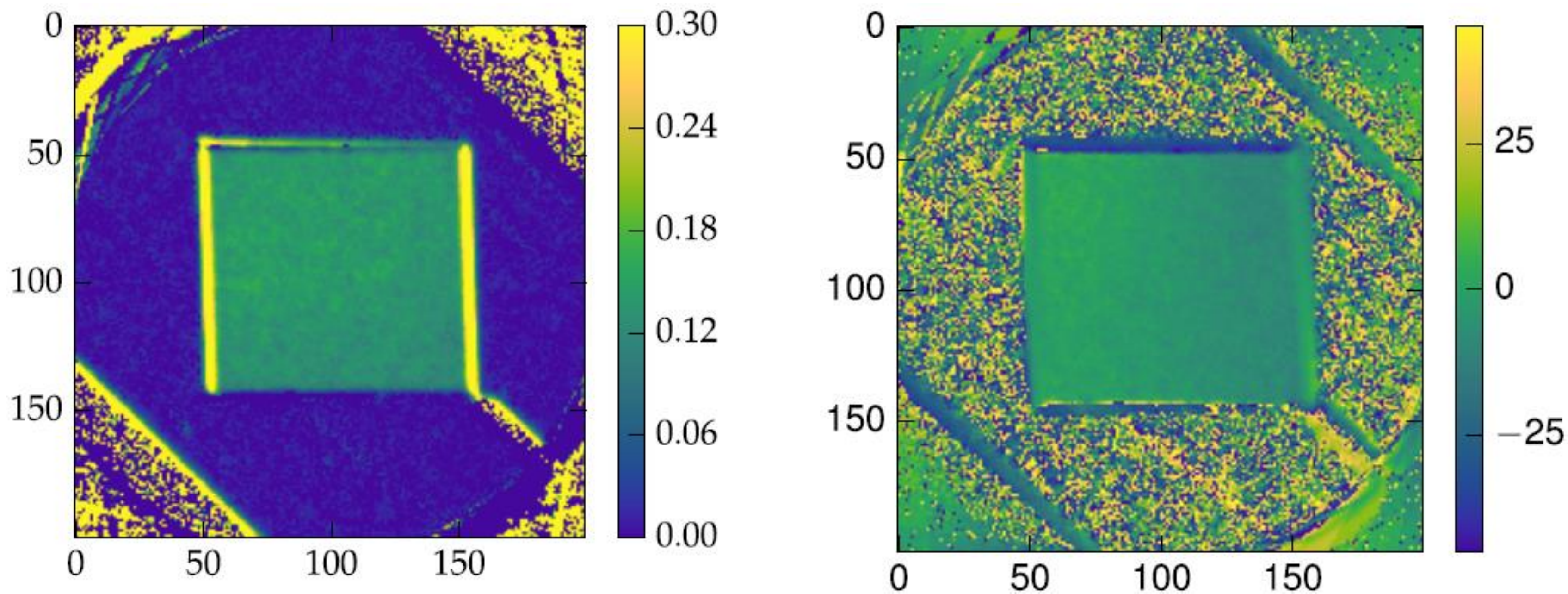
$\boldsymbol{\sigma_{an}}$ $\boldsymbol{\varphi}$



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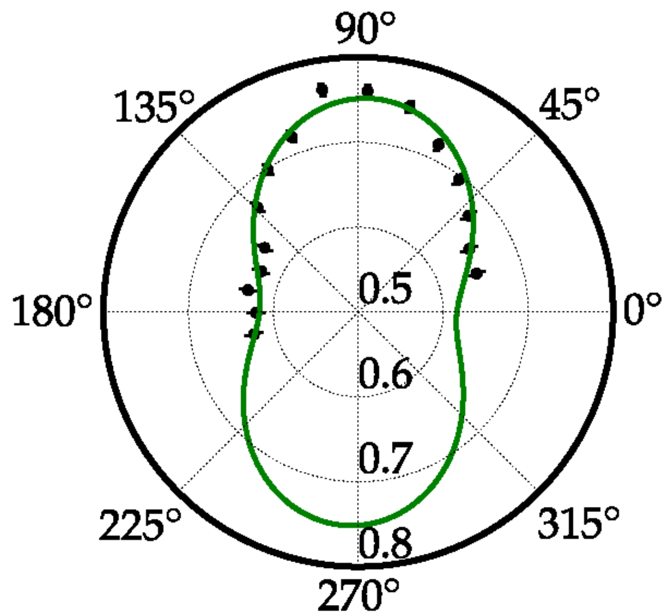
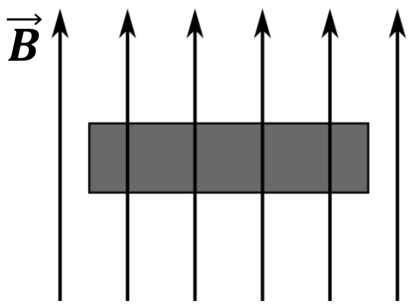
$\boldsymbol{DFI(\omega)}$



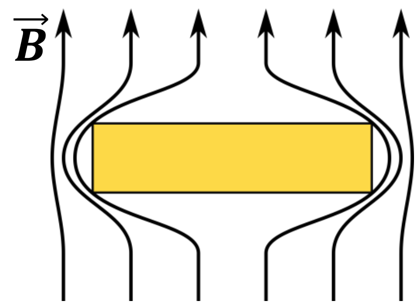
T. Neuwirth, Master Thesis (2017)

Normal Conductor



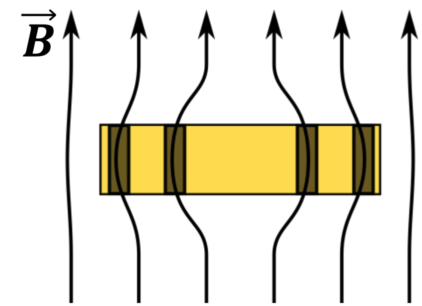
Superconductor

low field



Meissner state

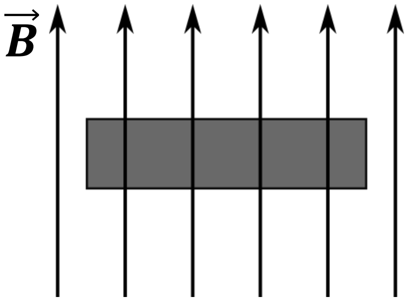
high field



Schubnikov state

Magnetic Domains in Superconductors

Normal Conductor



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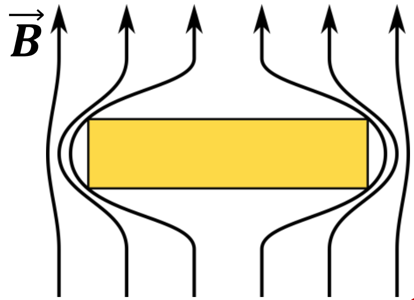
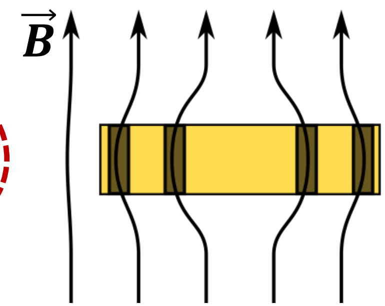
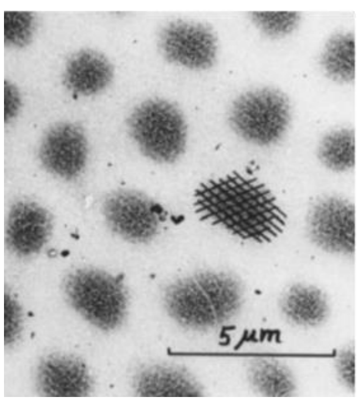


high field

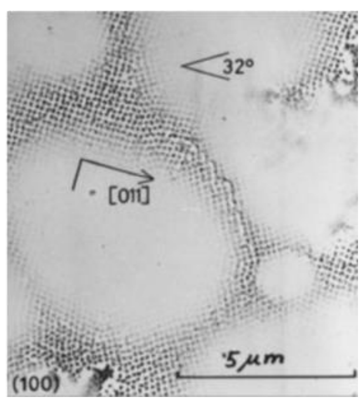


intermediate mixed state

bubble structure

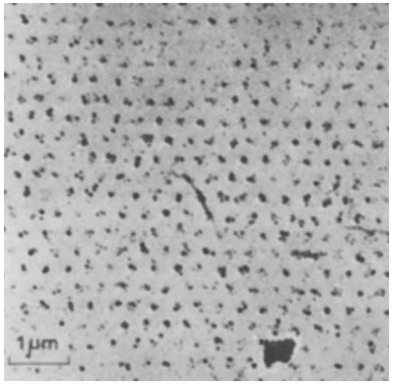


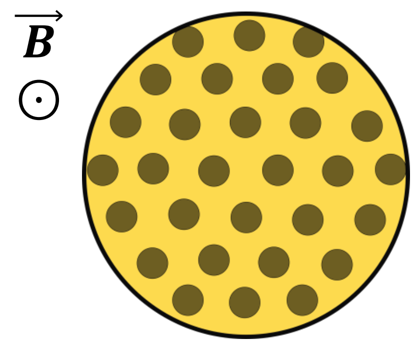
laminar structure



E. H. Brandt & M. P. Das, J. Supercond. Nov. Magn. (2011)

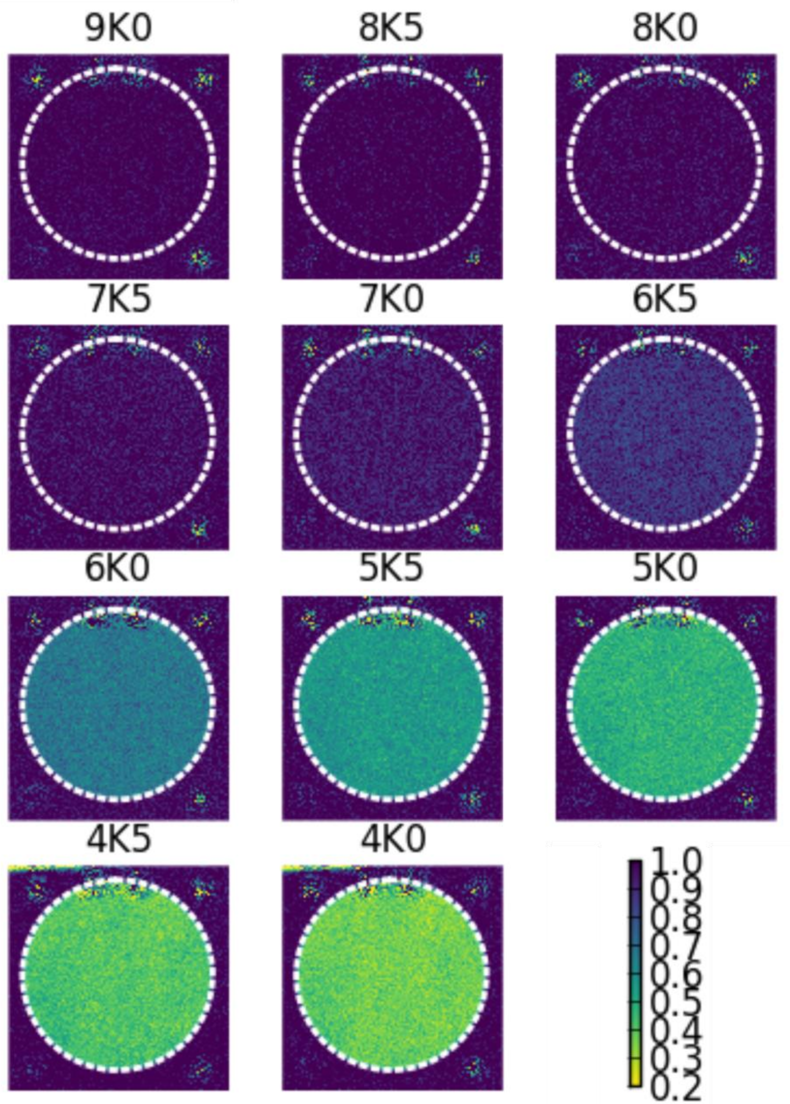
flux line lattice



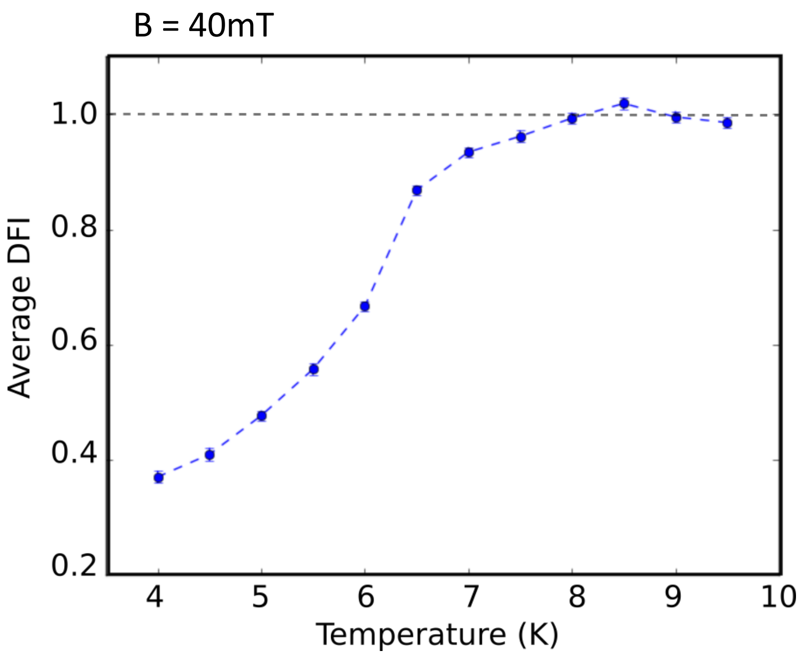


U. Essmann & H. Träuble, Phys. Lett. A (1967)

The Intermediate Mixed State in Niobium



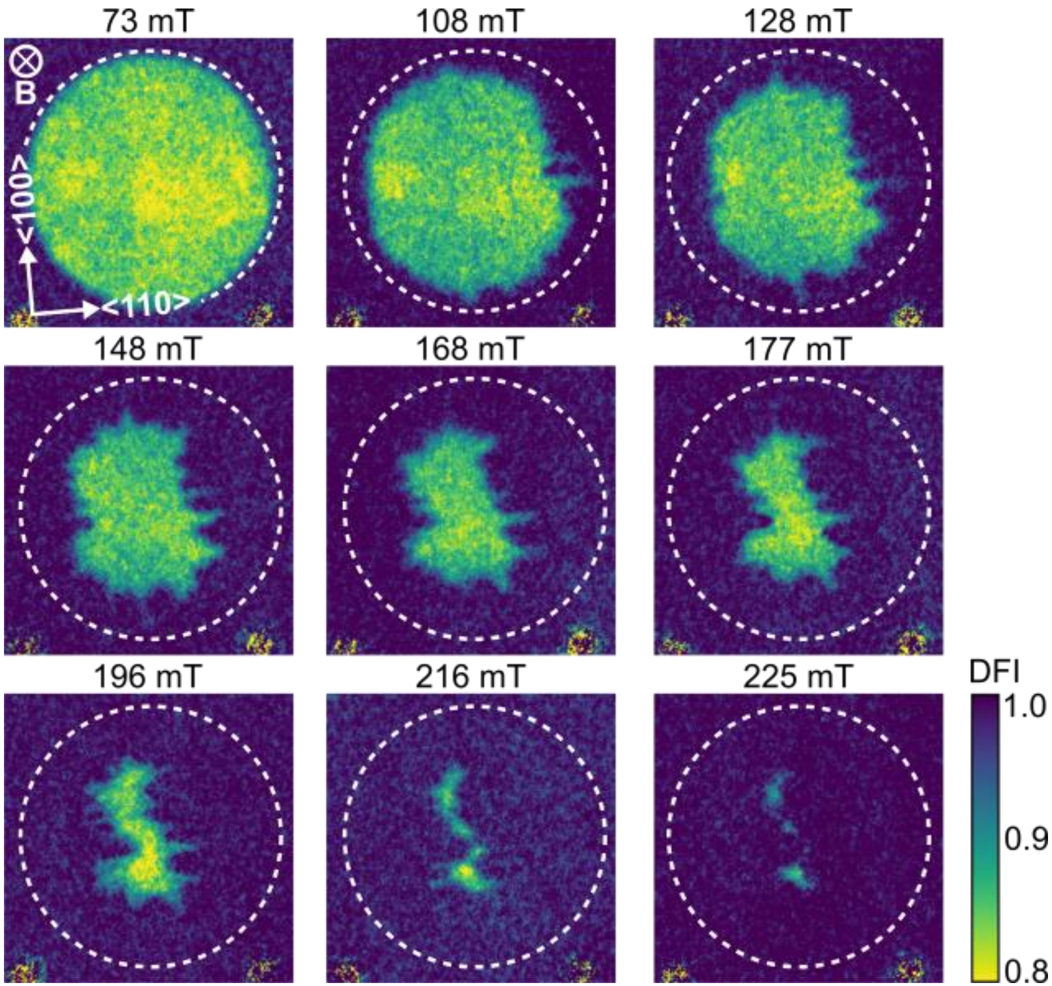
Field Cooling into the IMS



What changes?

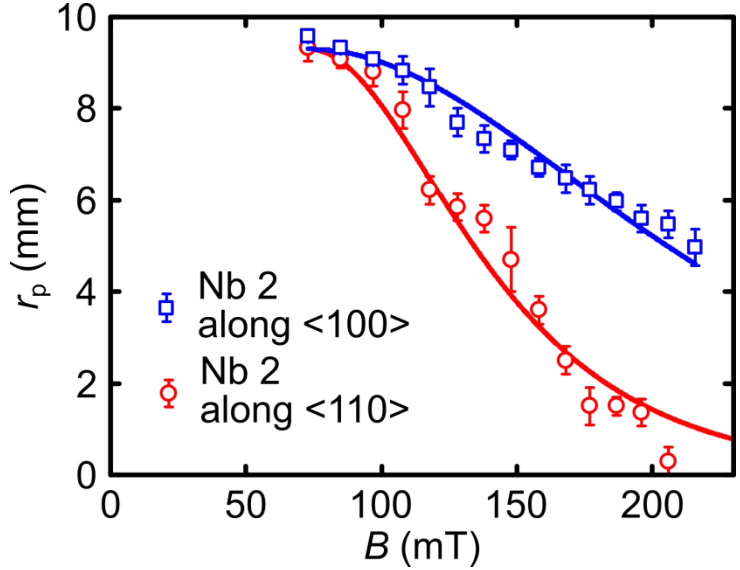
- domain size
- domain density
- scattering contrast

The Intermediate Mixed State in Niobium



T. Reimann, Ph.D Thesis (2016)

Supression of the IMS

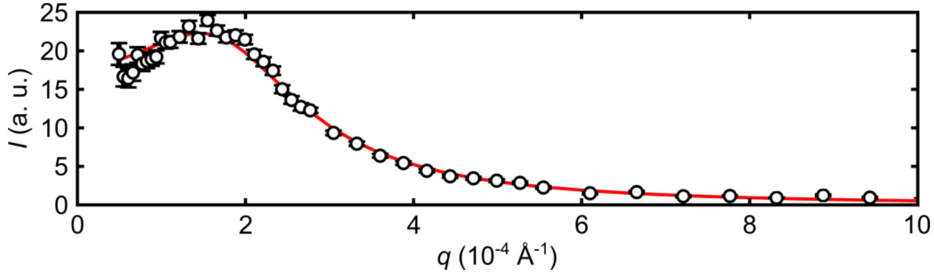


Additional field enters the sample
- IMS in the center
- FLL at the edge
- pinning along crystal axes

Heterogeneous Phase transition

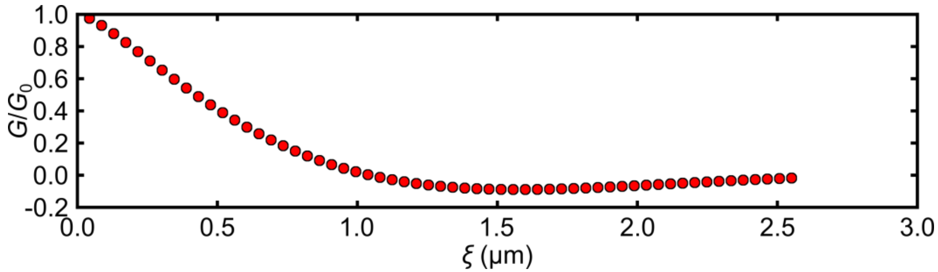
From USANS to nGI

USANS Measurement



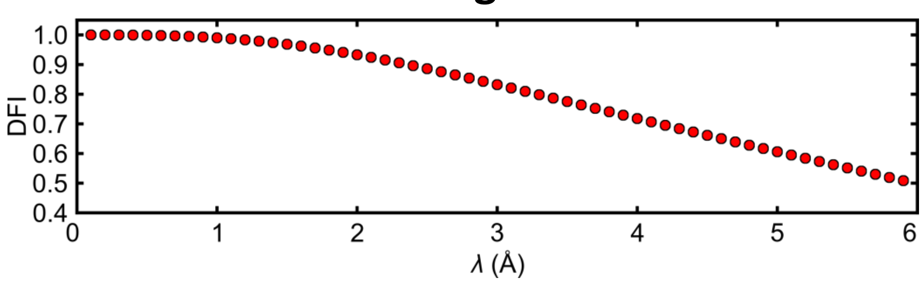
$$\left(\frac{d\sigma(q_x)}{d\Omega}\right)_{slit}$$

Pair Correlation Function



$$G(x,0)$$

DFI Signal



$$DFI = exp\left[\Sigma t\left(\frac{G(\xi_{GI})}{G(0)} - 1\right)\right]$$

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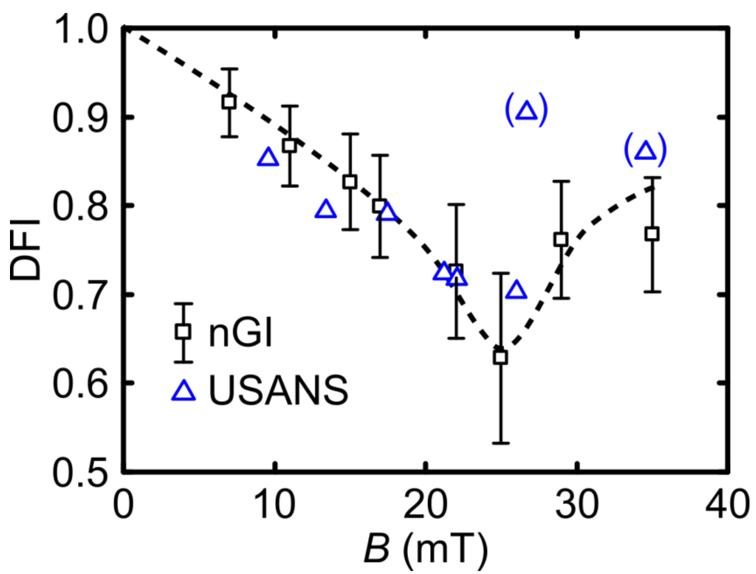
From USANS to nGI

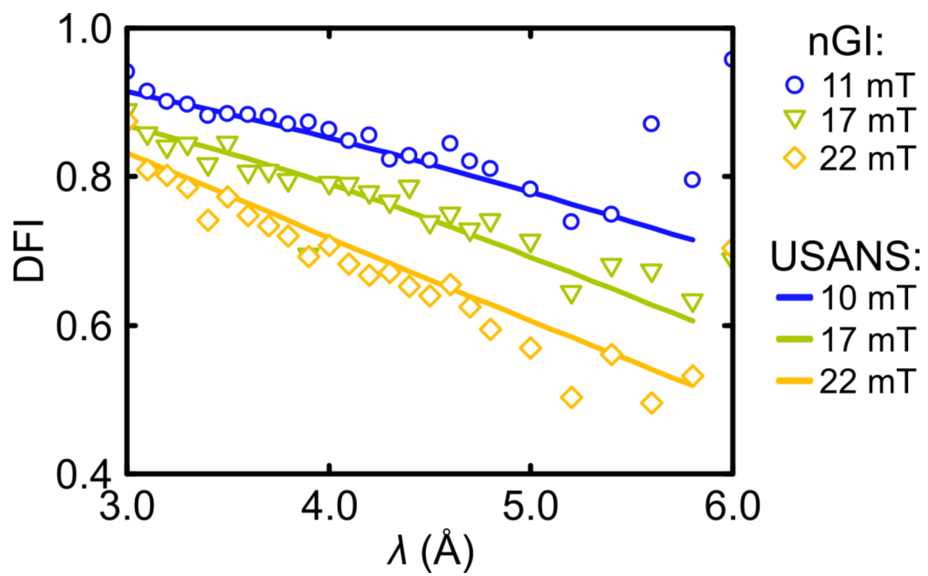
Magnetic Field Scan

Domain size changes
(USANS result)



Wavelength Scan

Probed ξ_{GI} changes



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